# PHARE: Modeling planetary magnetospheres with Adaptive Mesh and Model Refinement

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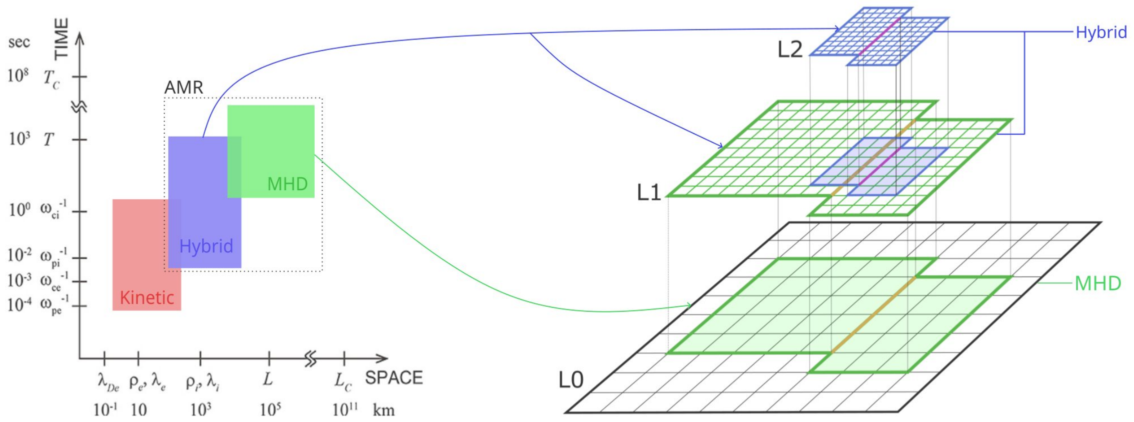
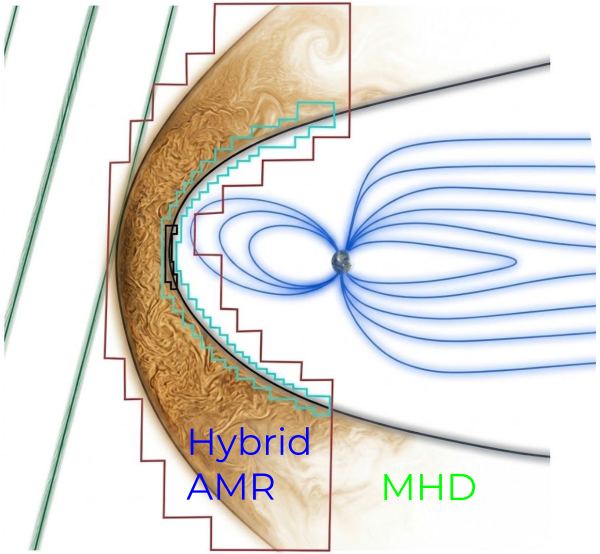
# Our goal in PHARE: adaptive mesh and model refinement (AM2R)




PHARE: an open-source, modern C++ plasma simulation code developed at LPP **for the community**.

# The equations we solve

We advance the following equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0$$

Continuity equation

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \left( \rho \mathbf{v}\mathbf{v} - \frac{\mathbf{B}\mathbf{B}}{\mu_0} + P^* \right) = 0$$

Momentum equation

$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \left[ \left( \mathcal{E} + P^* - \frac{B^2}{\mu_0} \right) \mathbf{v} + \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right] = 0$$

Energy equation

Polytropic closure:

$$\mathcal{E} = \frac{P}{\gamma - 1} + \frac{\rho v^2}{2} + \frac{B^2}{2\mu_0}$$

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$$\frac{\partial \mathcal{E}}{\partial t} + \nabla \cdot \left[ \left( \mathcal{E} + P^* - \frac{B^2}{\mu_0} \right) \mathbf{v} + \frac{\mathbf{E} \times \mathbf{B}}{\mu_0} \right] = 0$$

Energy equation

$$\frac{\partial \mathbf{B}}{\partial t} + \nabla \times \mathbf{E} = 0$$

Faraday's law

$$\mathbf{E} = -\mathbf{v} \times \mathbf{B} + \eta \mathbf{j} + \frac{\mathbf{j} \times \mathbf{B}}{e n_e}$$

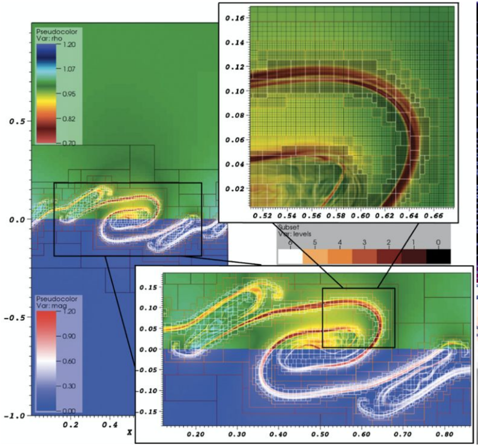
Ohm's law

$$\mathbf{j} = \frac{\nabla \times \mathbf{B}}{\mu_0}$$

Ampere's law

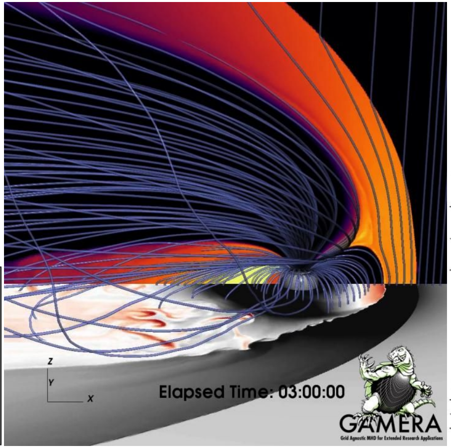
# State of the art of space plasmas MHD codes

PLUTO



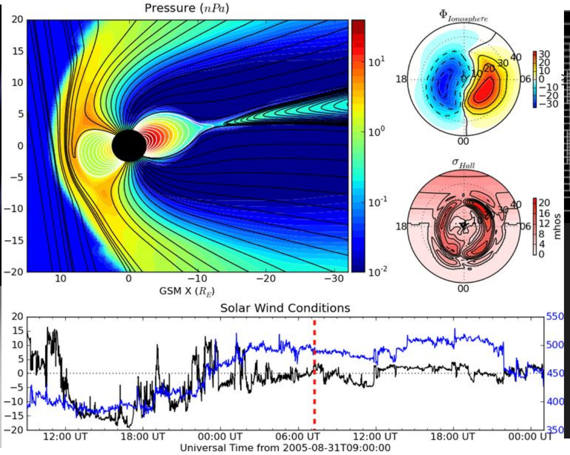

[Mignone et al. 2011]

GAMERA



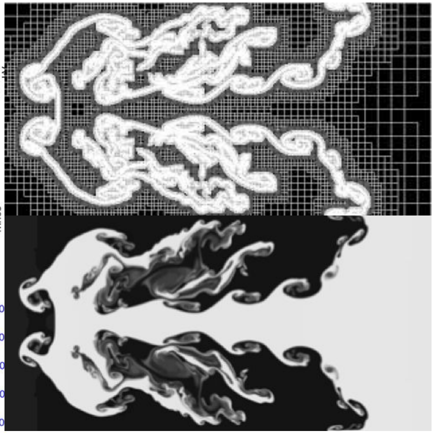

[Zhang et al. 2019]

BATS-R-US




[Gombosi et al. 2003]

RAMSES

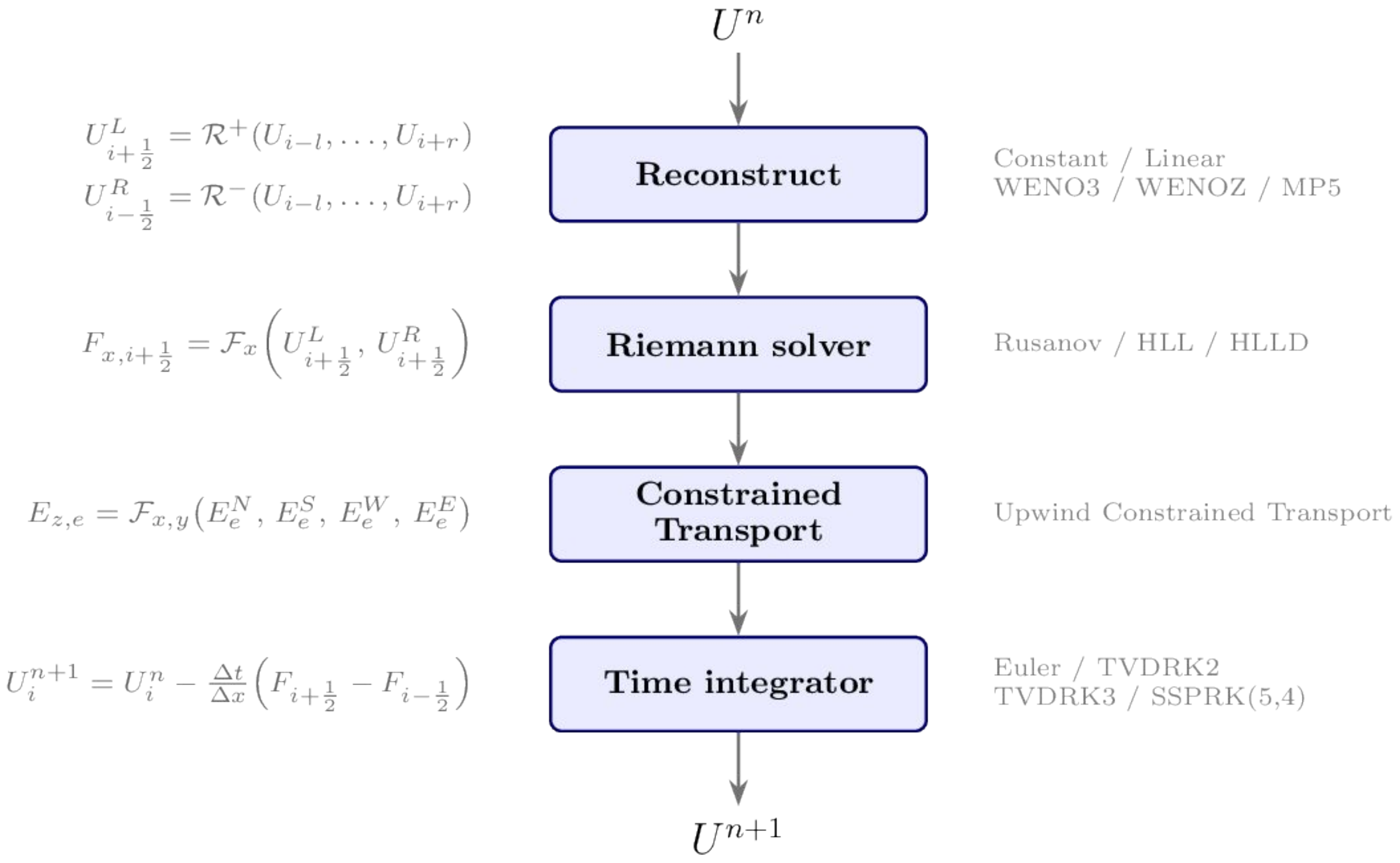


[Teyssier et al. 2002]

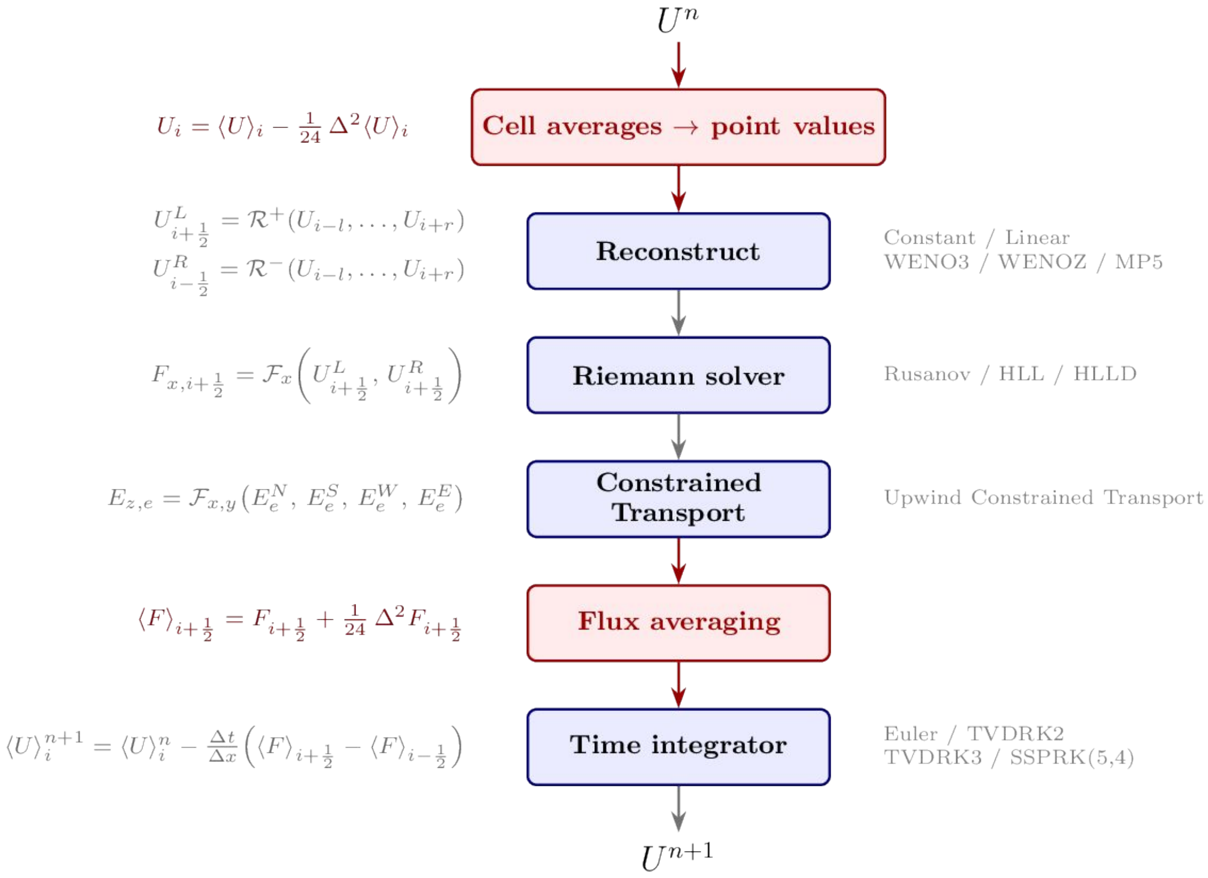
\+ Athena, Idefix, etc...

- Godunov finite volumes widely adopted by the community because, conservative, good at capturing shock, robust.
- To our knowledge, no higher than second order codes with AMR and Hall MHD.
- High order interesting for Hall because: $\Delta t \sim \Delta x^2$ $\quad \tau = \mathcal{O}(\Delta t^{p+1})$

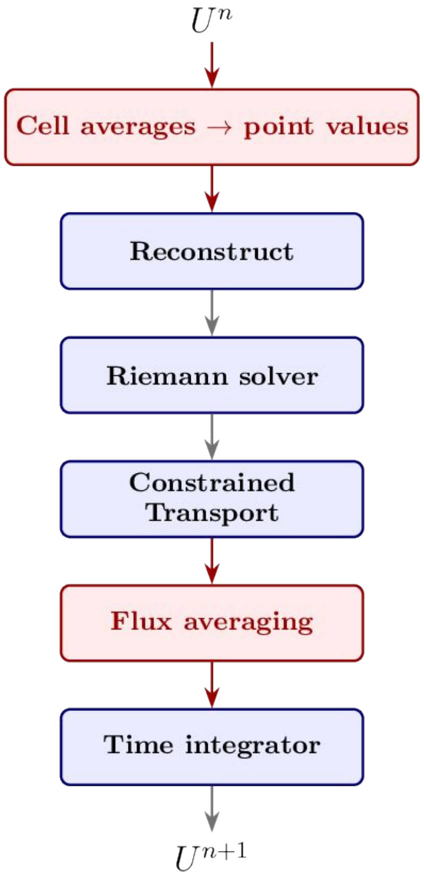
# Overview of Godunov-type finite volume algorithm

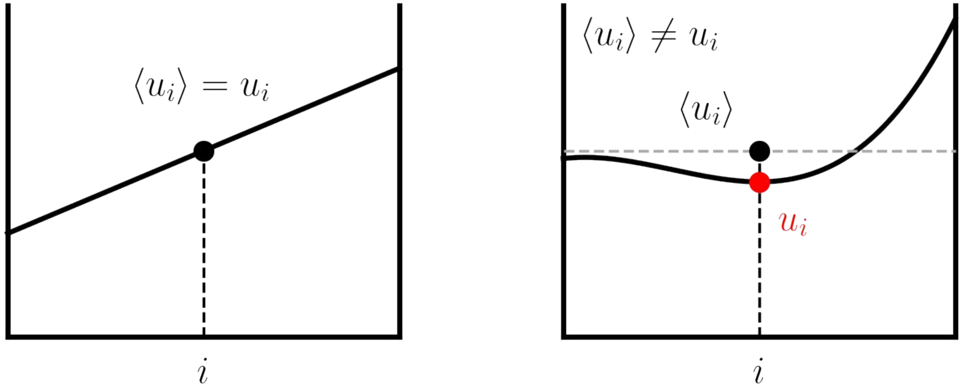


# The 4th order algorithm

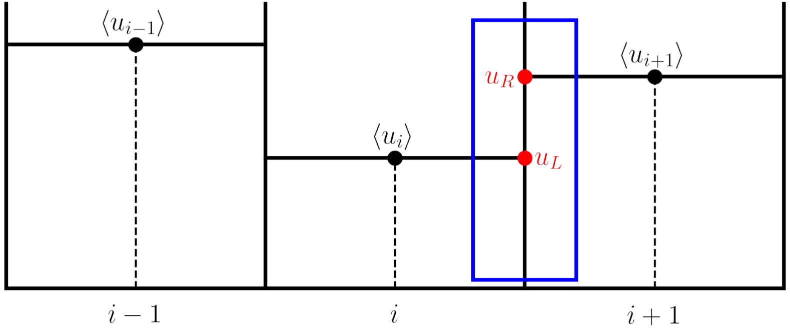


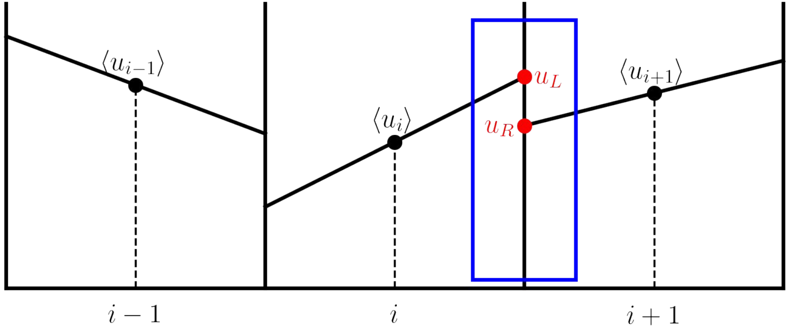
# The 4th order algorithm

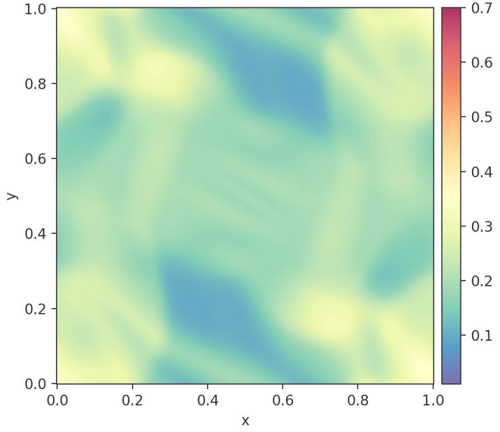


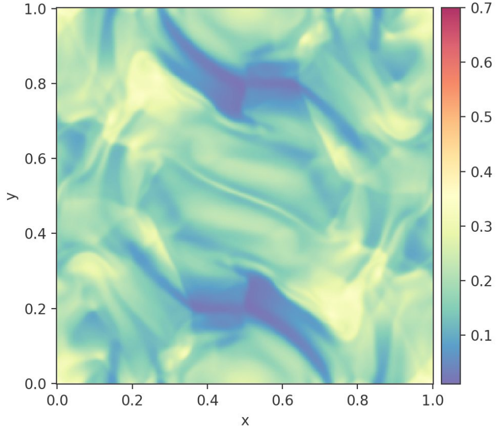



# Reconstruction



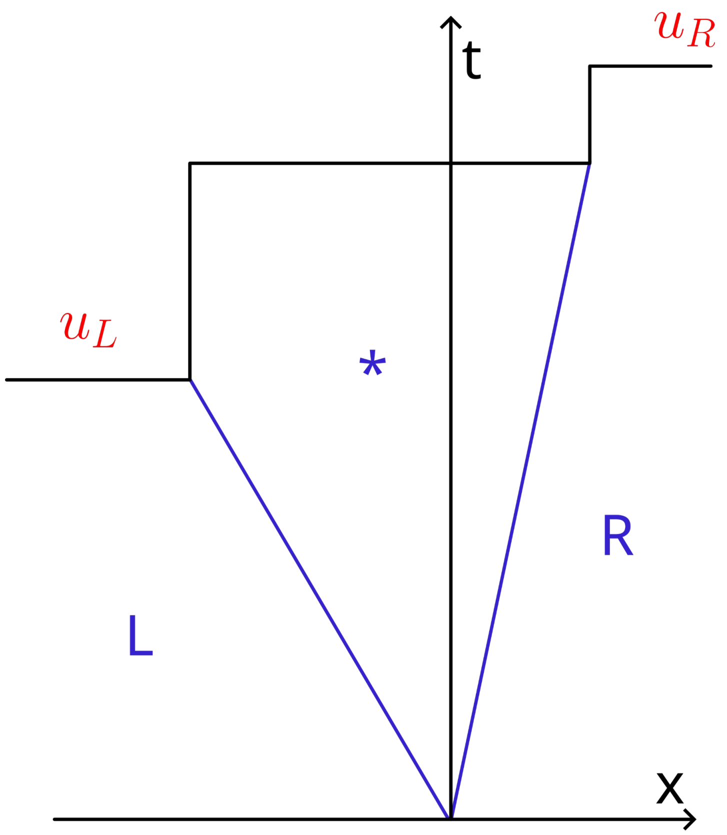
# Riemann solver

$$\partial_t U + \nabla \cdot F(U) = 0$$

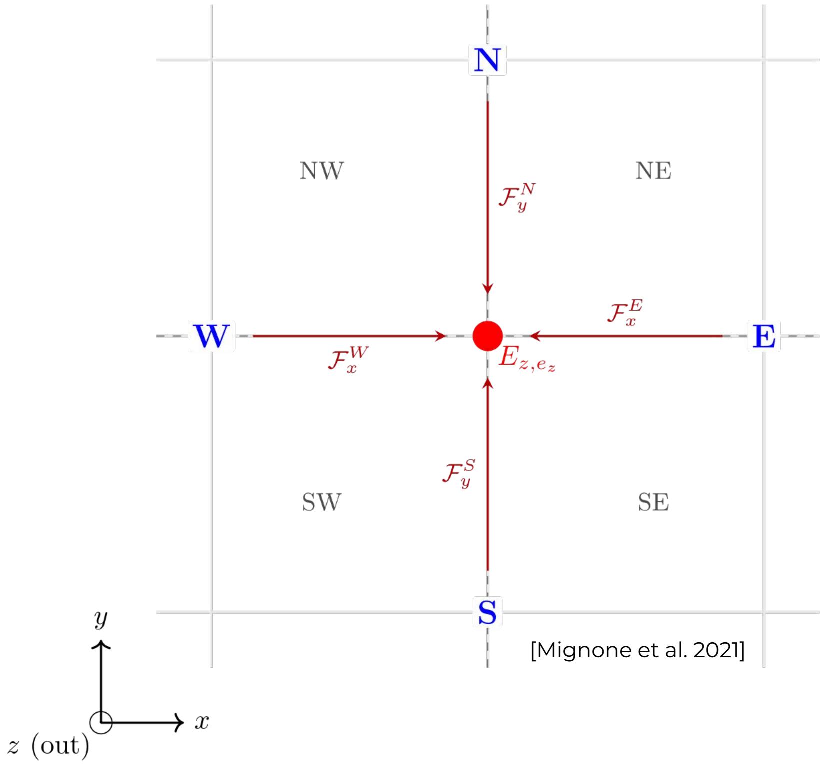
$$U(x,0) = \begin{cases} U_L & x < 0 \\ U_R & x > 0 \end{cases}$$

$$\hat{F}(U_L, U_R) = F(U_\alpha)$$

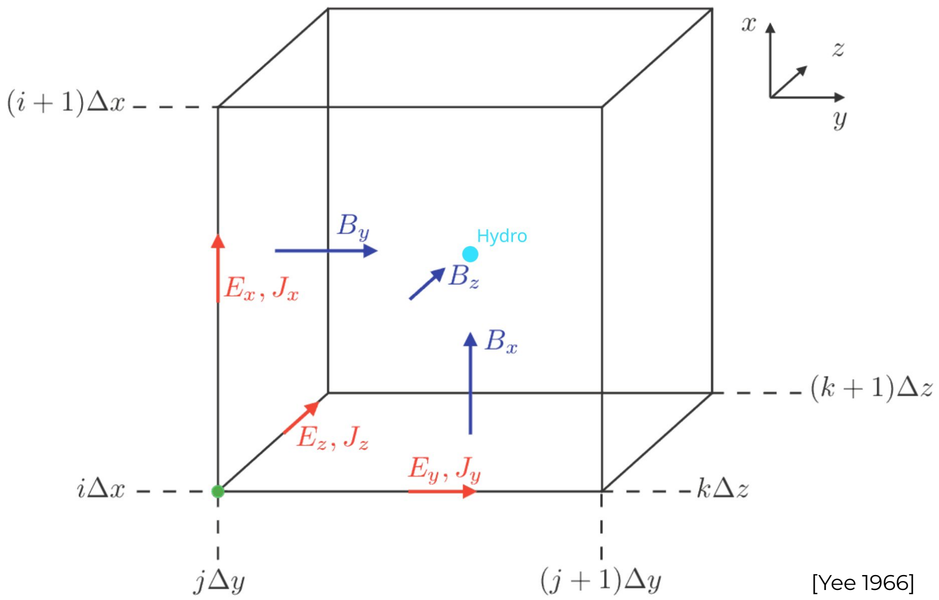
$$\alpha \in \{L,\, *,\, R\}$$



# Constrained Transport



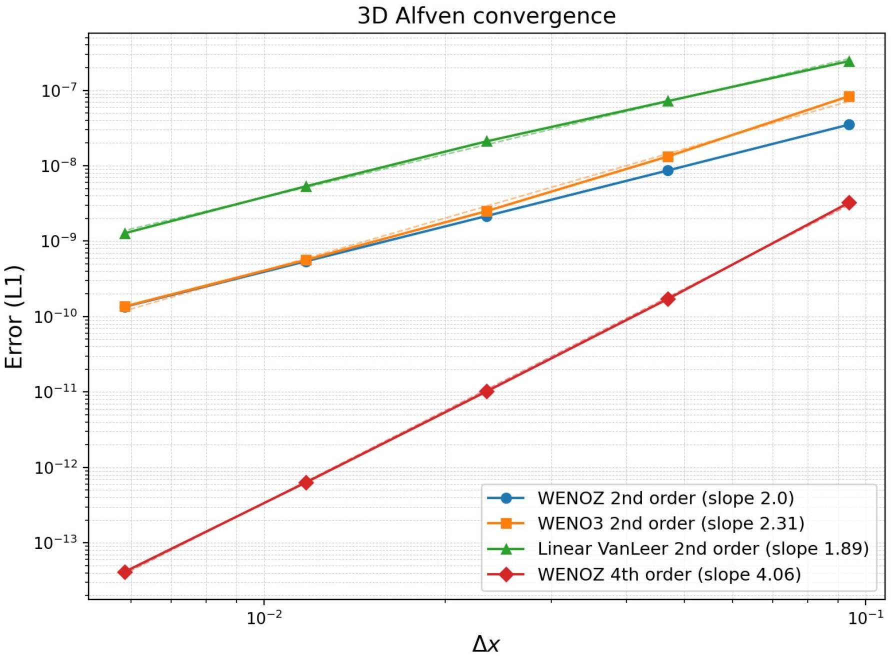

[Mignone et al. 2021]

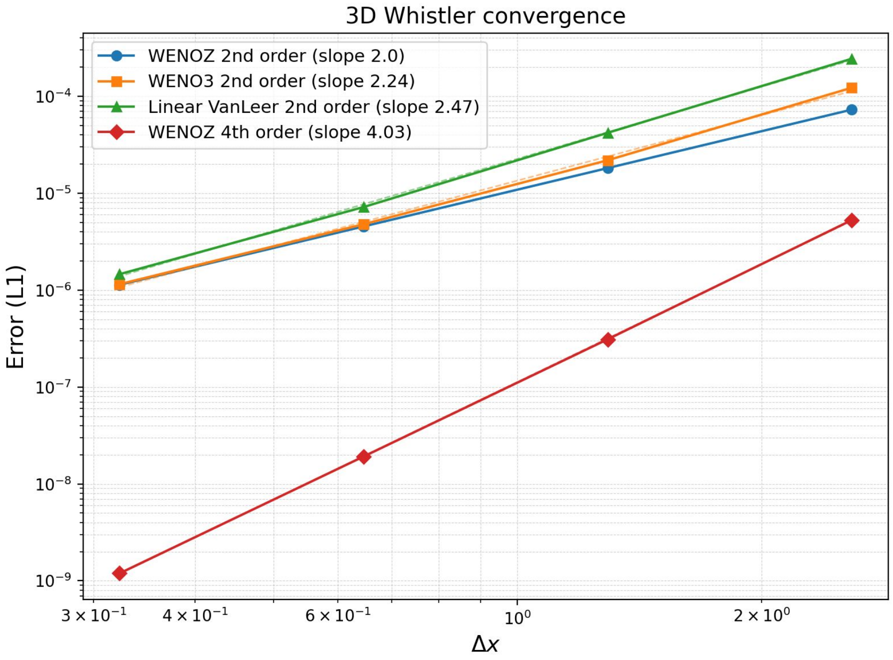



[Yee 1966]

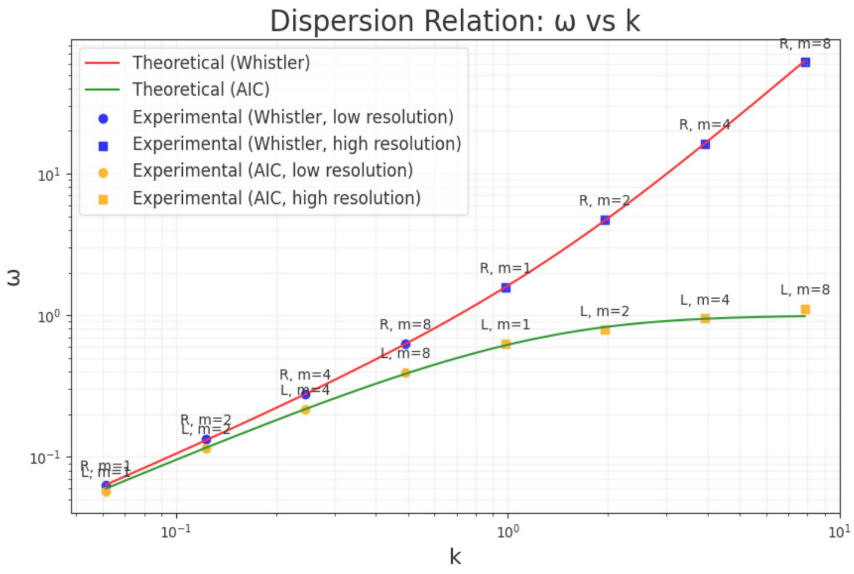
Reuse information from the hydro 1d Riemann solvers to approximate a 2d Riemann solve on the edge.

# Theoretical validation: convergence test



# Theoretical Validation: Hall MHD dispersion




Alfvén Ion Cyclotron:

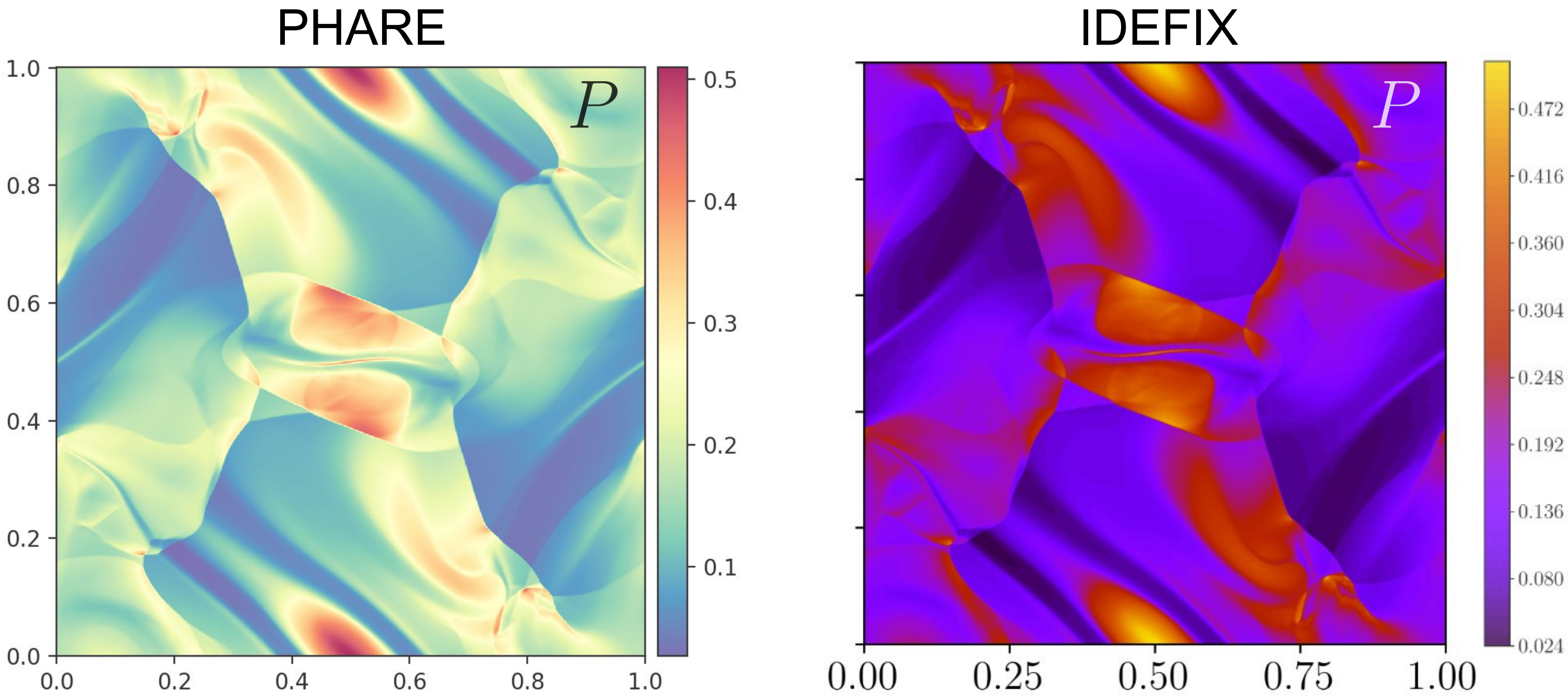
$$\omega_L = \frac{k^2}{2}\left(\sqrt{1+\frac{4}{k^2}} - 1\right)$$

Whistler:

$$\omega_R = \frac{k^2}{2}\left(\sqrt{1+\frac{4}{k^2}} + 1\right)$$

# Functional Test: Orszag-Tang vortex

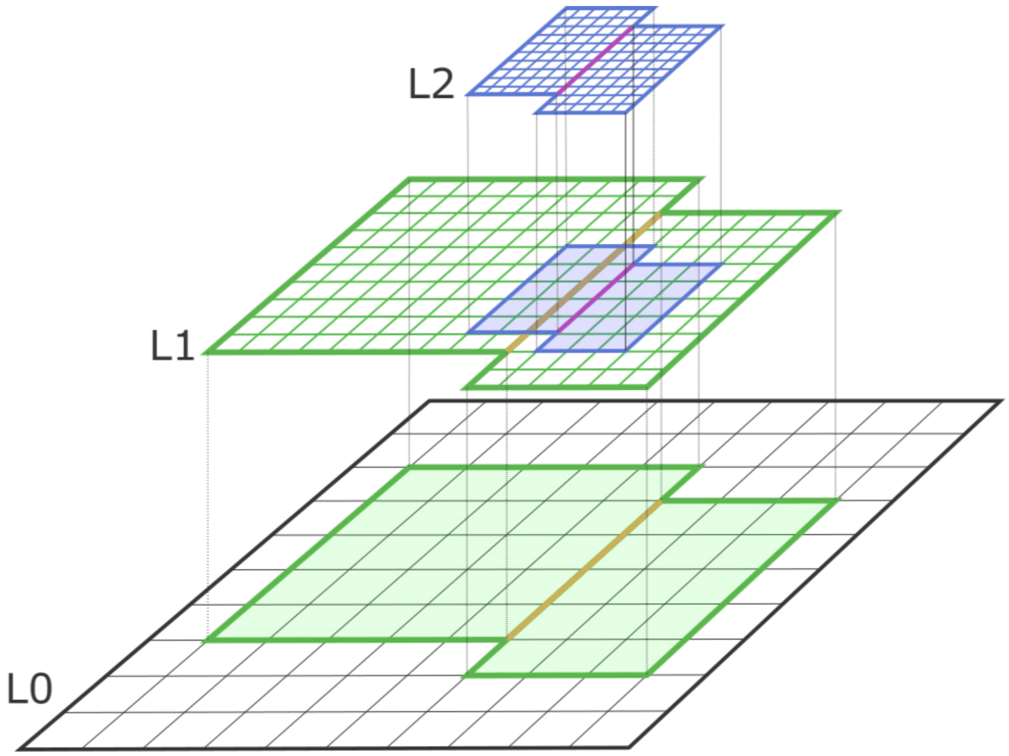
[Orszag & Tang 1979]

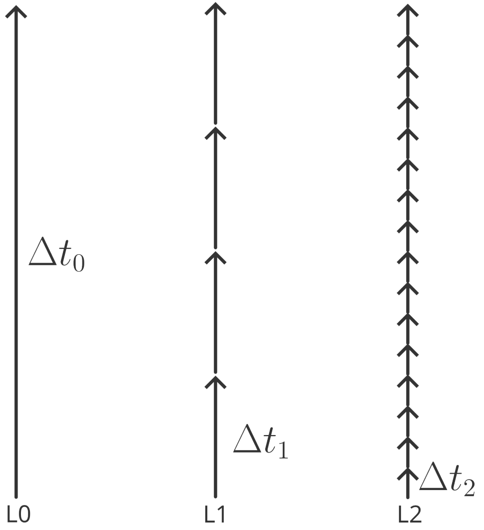



Pressure plot of the Orszag-Tang vortex at t=0.5 with a domain of 1, 1024x1024 cells and a second order scheme. PHARE (left) Idefix (right).

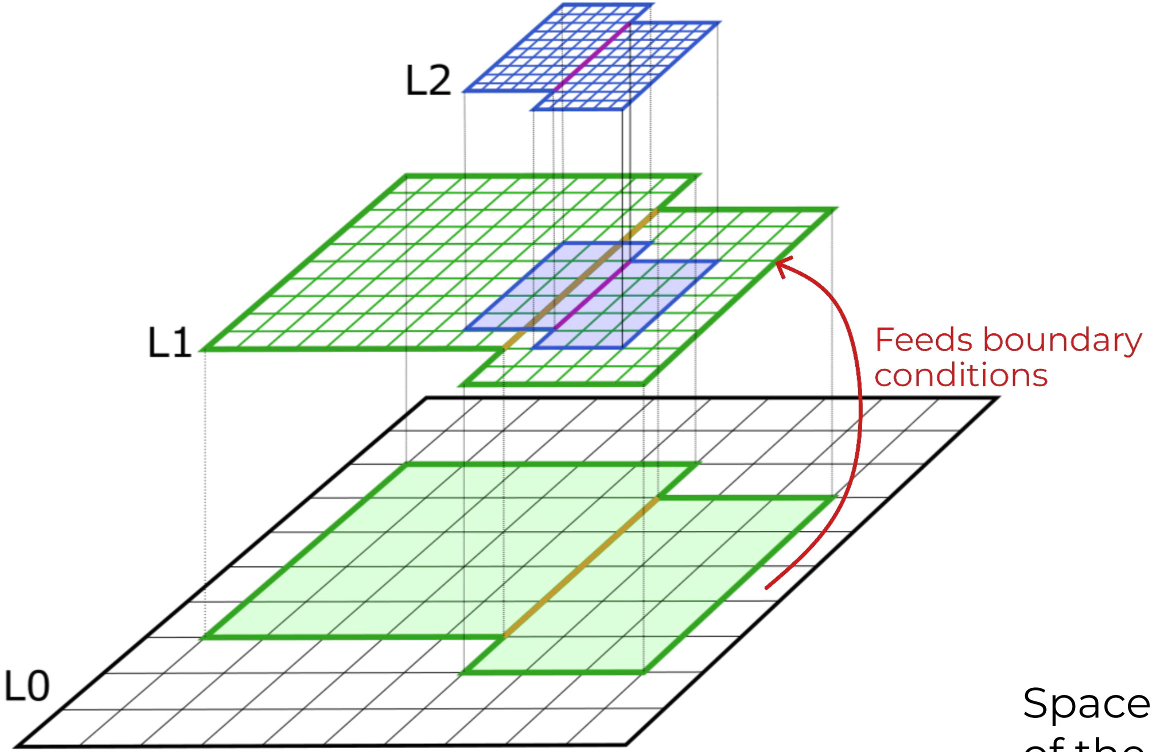
code IDEFIX: [Lesur et al. 2023]

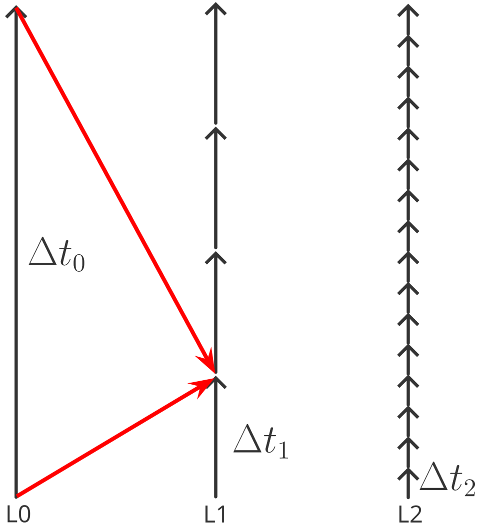
# AMR Hierarchy



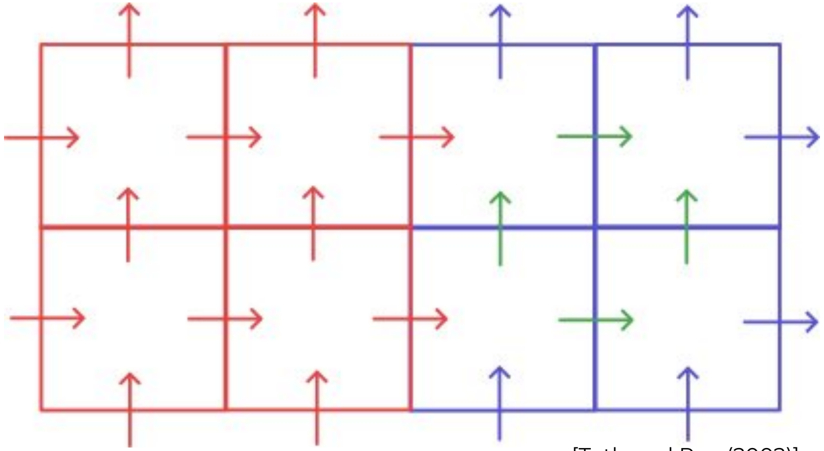
# Refinement: feeding boundary conditions




Space and time interpolation of the data at boundaries.

# The conservative refinement problem



[Toth and Roe (2002)]

Idea: refine the new fine faces in such a way that:

$$\nabla \cdot \mathbf{B}_{fine} = \frac{1}{r^d} \nabla \cdot \mathbf{B}_{coarse}$$

with $r$ the refinement ratio and $d$ the dimension.

$$u = B_x, v = B_y, w = B_z$$

$$u^{0,j,k} = \tfrac{1}{2}(u^{+,j,k} + u^{-,j,k}) + U_{xx} + k(\Delta z)^2 V_{xyz} + j(\Delta y)^2 W_{xyz}$$

$$v^{i,0,k} = \tfrac{1}{2}(v^{i,+,k} + v^{i,-,k}) + V_{yy} + i(\Delta x)^2 W_{xyz} + k(\Delta z)^2 U_{xyz}$$

$$w^{i,j,0} = \tfrac{1}{2}(v^{i,j,+} + v^{i,j,-}) + W_{zz} + j(\Delta y)^2 U_{xyz} + i(\Delta x)^2 V_{xyz}$$

with:

$$U_{xx} = \tfrac{1}{8} \textstyle\sum_{ijk=\pm} ijv^{i,j,k} + ikw^{i,j,k}$$

$$V_{yy} = \tfrac{1}{8} \textstyle\sum_{ijk=\pm} iju^{i,j,k} + jkw^{i,j,k}$$

$$W_{zz} = \tfrac{1}{8} \textstyle\sum_{ijk=\pm} iku^{i,j,k} + jkv^{i,j,k}$$

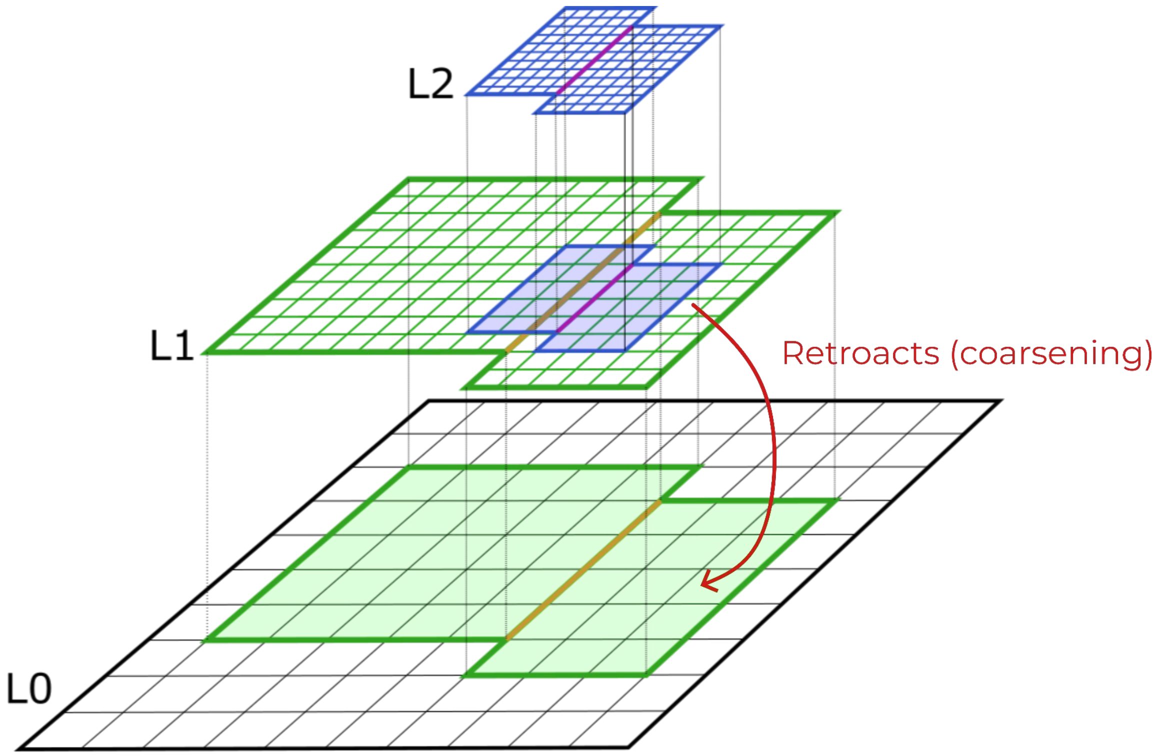
and:

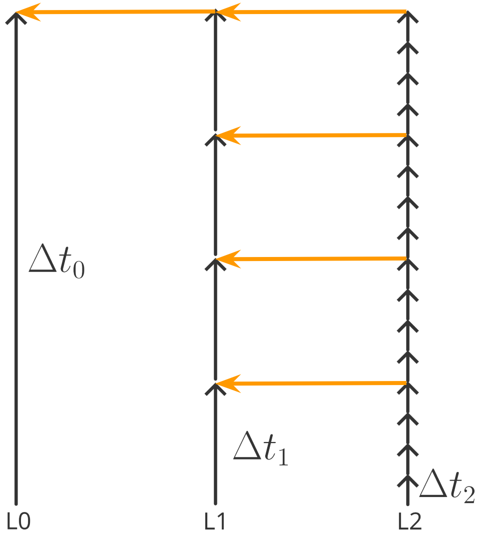
$$U_{xyz} = \tfrac{1}{8} \textstyle\sum_{ijk=\pm} \frac{ijku^{i,j,k}}{(\Delta y)^2+(\Delta z)^2}$$

$$V_{xyz} = \tfrac{1}{8} \textstyle\sum_{ijk=\pm} \frac{ijkv^{i,j,k}}{(\Delta x)^2+(\Delta z)^2}$$

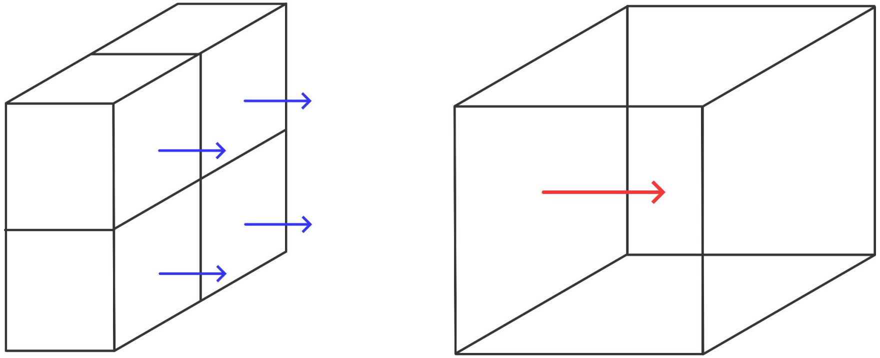
$$W_{xyz} = \tfrac{1}{8} \textstyle\sum_{ijk=\pm} \frac{ijkw^{i,j,k}}{(\Delta x)^2+(\Delta y)^2}$$

# Retroaction of the fine fields



# Maintaining conservation at coarse-fine boundaries



Need to express the whole time integration step with a single flux. With multistep integrators, use Butcher flux:

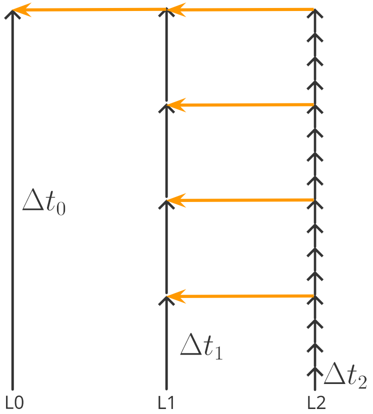
$$\mathbf{U}^{n+1} = \mathbf{U}^{n} + \sum_{step} \gamma(\omega_{step}, \omega_{step-1}, \ldots, \omega_0)\mathbf{F}^{step}$$

$$\mathbf{F}^{total}_{fine} = \frac{1}{\mathcal{C}_{\text{sub-steps}} \times \mathcal{C}_{\text{shared fine faces}}} \sum_{\text{sub-steps}} \sum_{\text{shared fine faces}} \mathbf{F}_{fine}$$

← Space-time average of the fluxes

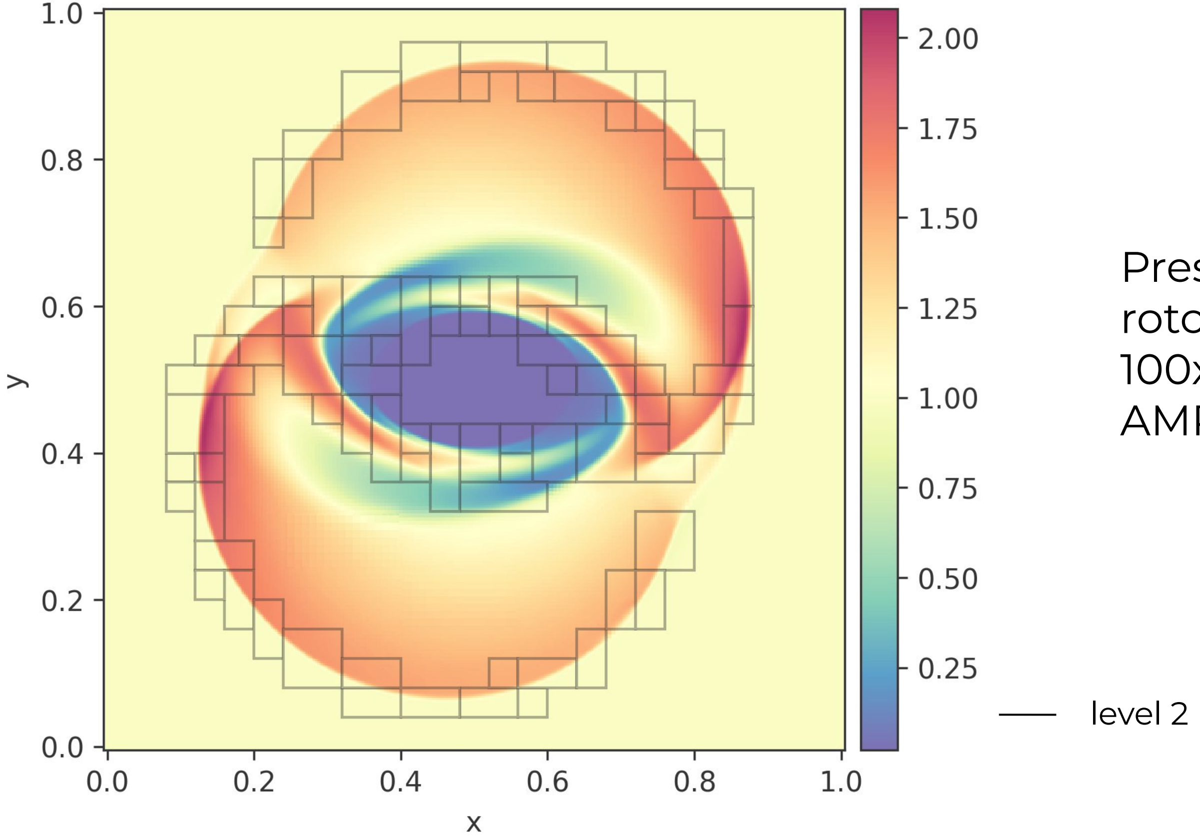
$$\mathbf{U}^{n+1} = \mathbf{U}^{n+1} - \frac{\Delta t_{coarse}}{\Delta l}\left(\mathbf{F}_{coarse} - \mathbf{F}^{total}_{fine}\right)$$

← Correction step



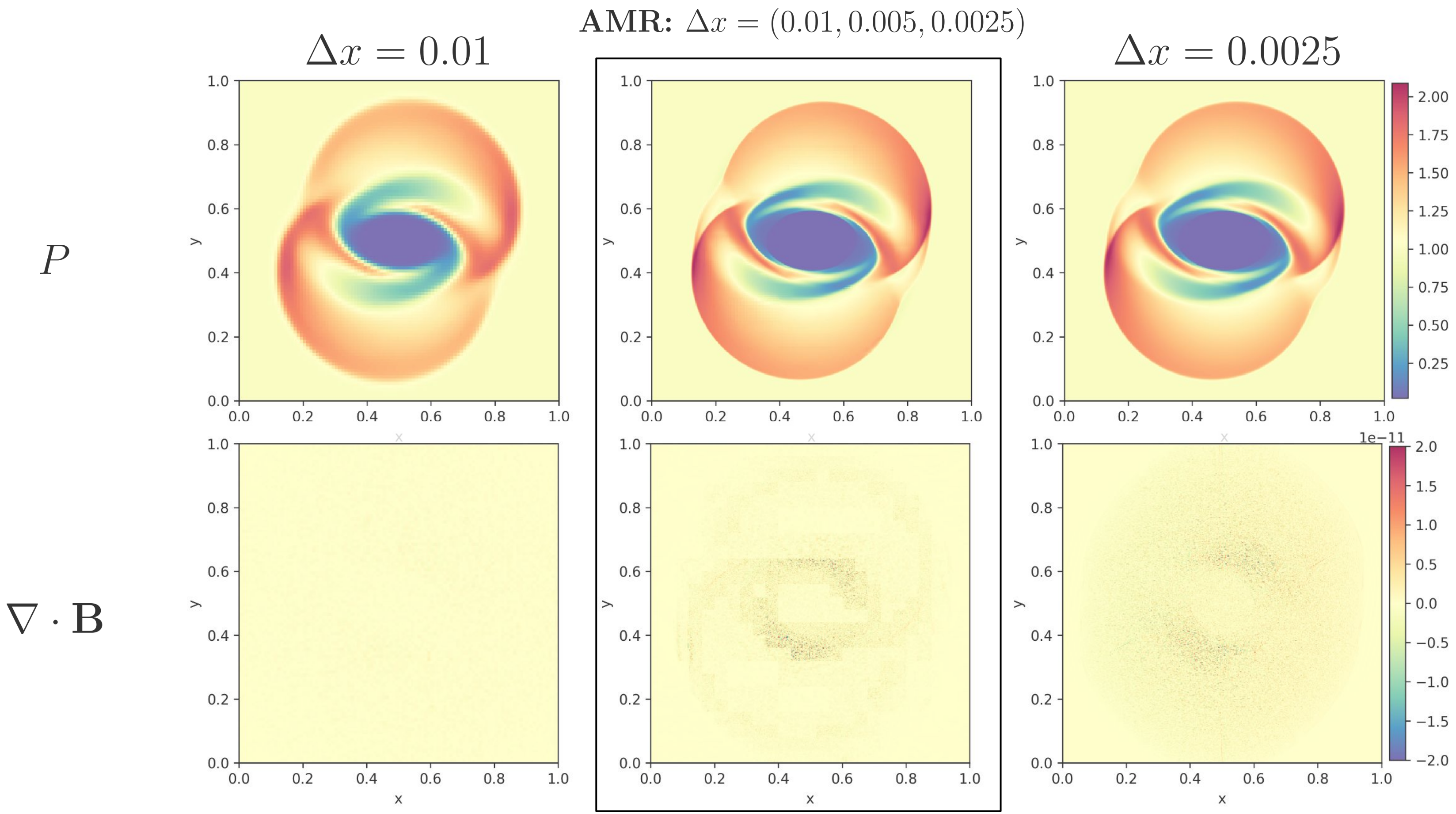
# AMR Validation: MHD Rotor test

[Balsara & Spicer 1999]




Pressure for the MHD rotor test at t=0.15. 100x100 grid points, 3 AMR level.

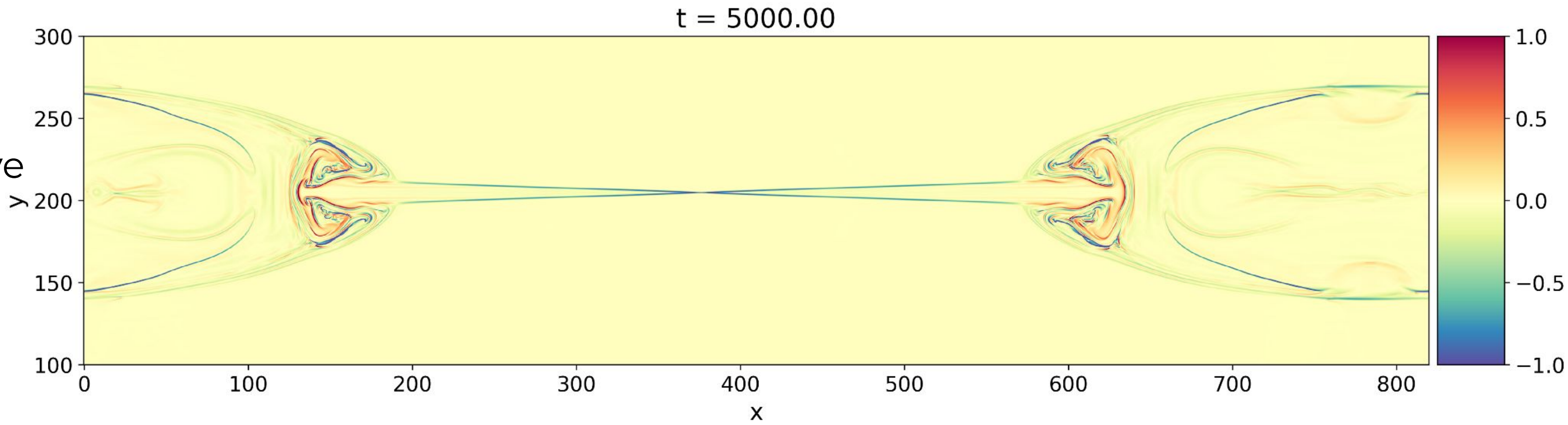
# AMR Validation: MHD Rotor test



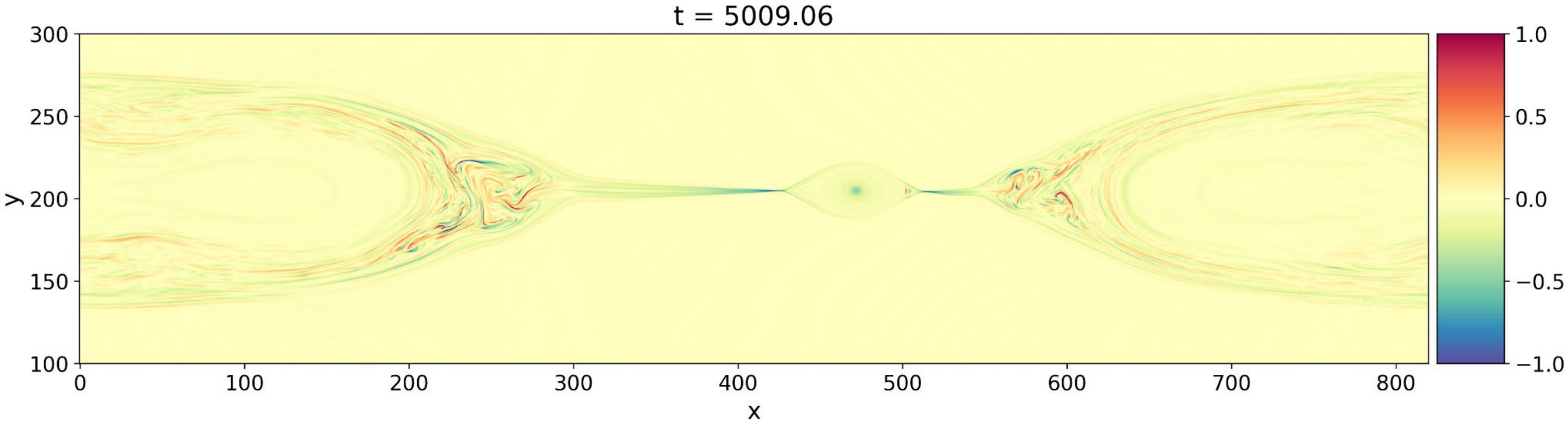
# High Lundquist simulations

$10^5$

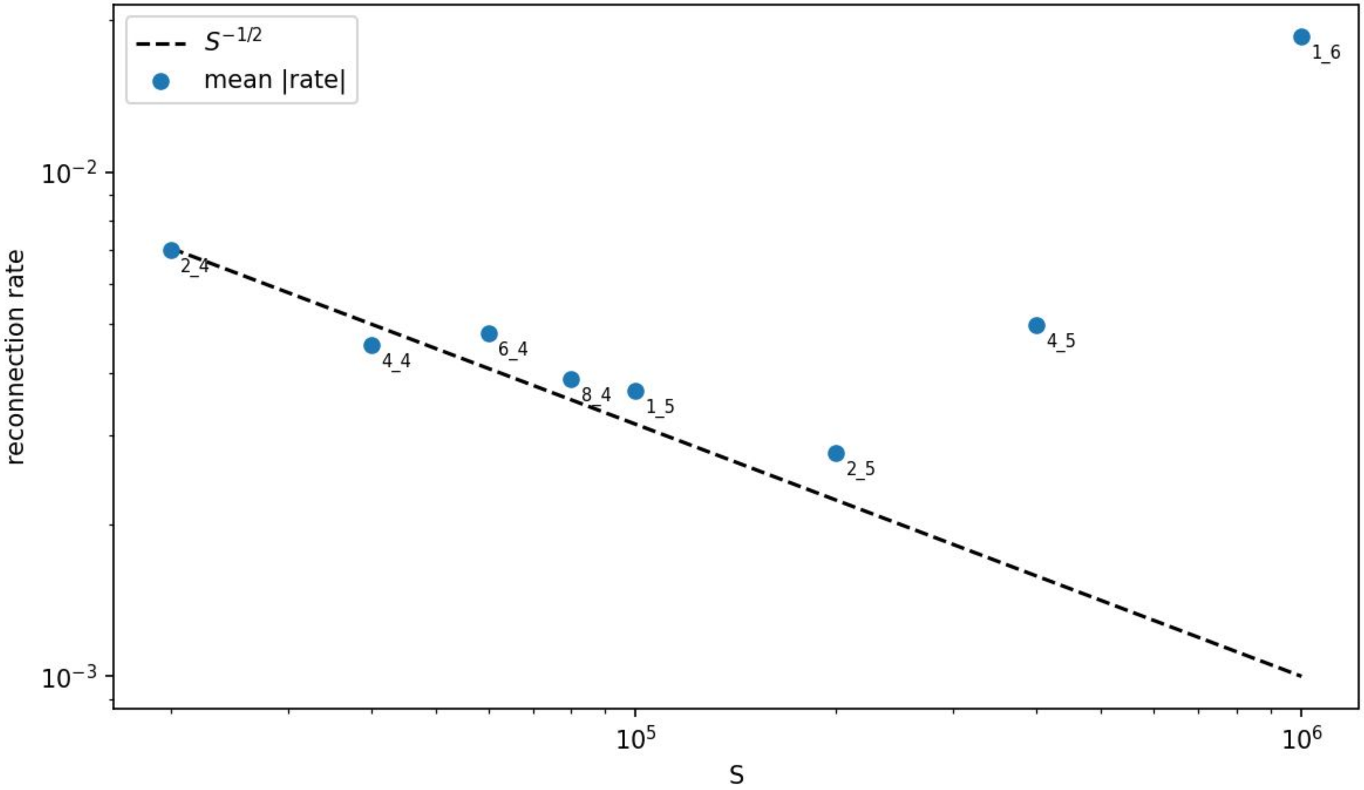
Resistive
MHD



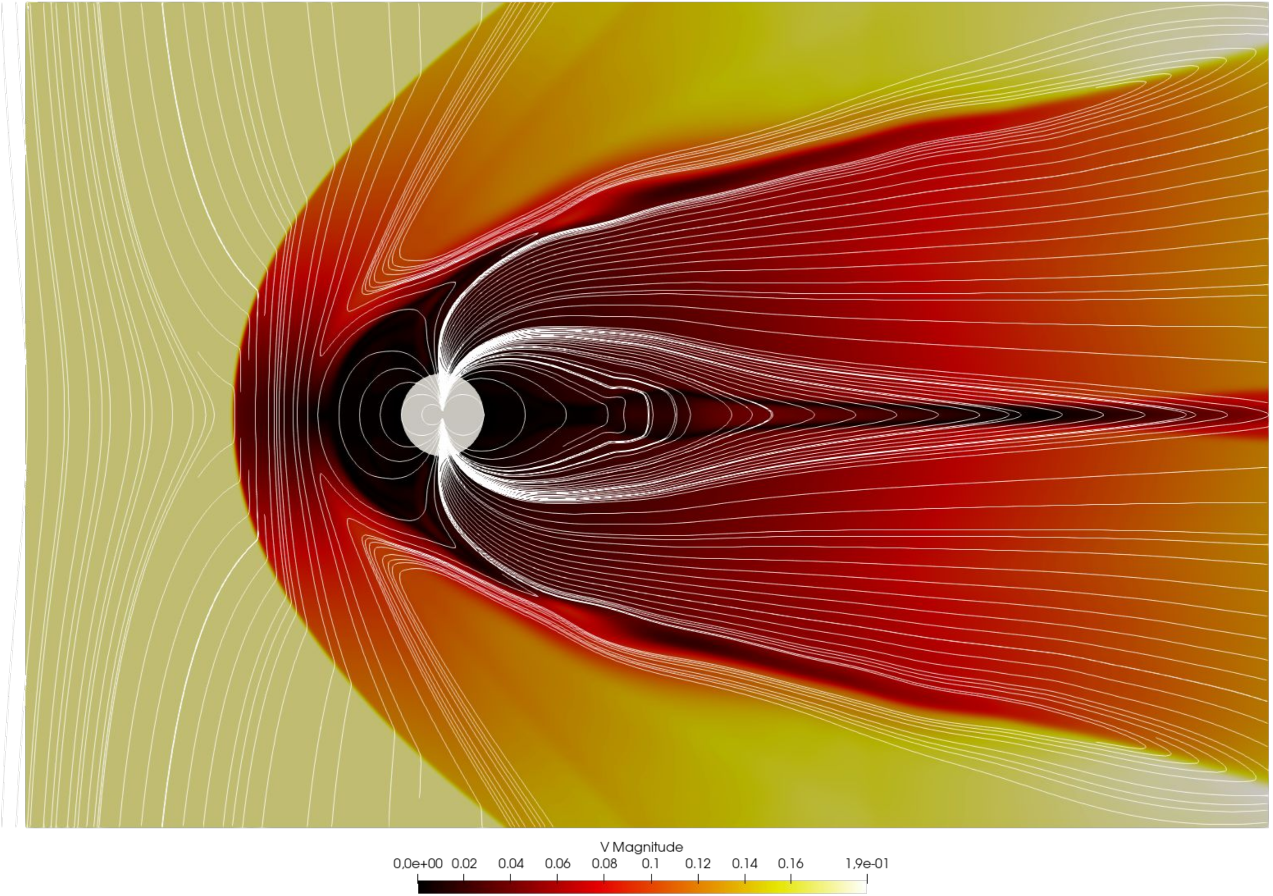

Hall
MHD



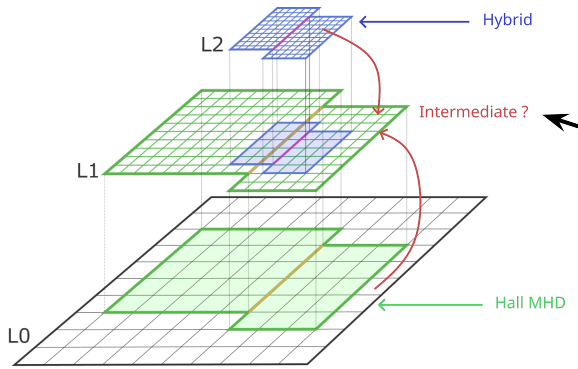
# Lundquist scaling

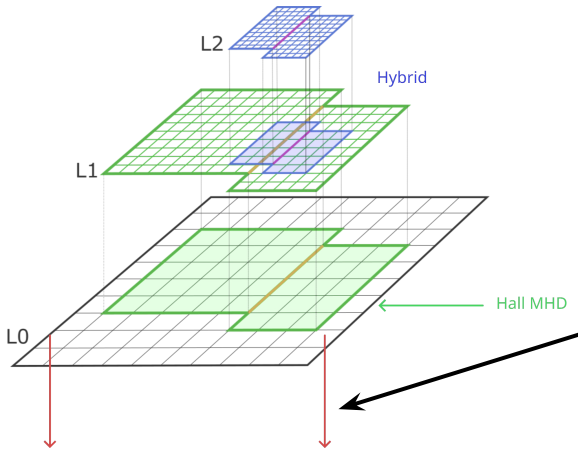


# First magnetospheric simulations



# Lots to experiment with on the coupling side




- Higher moment schemes ?

- Test particles in MHD ?
- Guiding center schemes ?

- Ideal MHD ?
- With asymptotic preserving schemes ?

# Next Steps

- Extend magnetospheric boundary conditions
- Improved performances
- MHD/Hybrid coupling

→ The main ingredients for our global simulations