A new method to approximate the guiding centre motion in particle tracing algorithms

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Context

Particle tracing algorithms are used in plasma research to infer the trajectory of particles in an electromagnetic field. Such algorithms are used to study tokamaks, magnetospheres, the solar wind and many more other cases. We present a new method, proposed by Xiao and Qin in 2021 [1], to approximate the guiding centre motion. The method has been tested for different magnetic field configurations and seems to be promising.

This method enables the use of time steps larger than the gyromotion of the particle and allows to get rid of the particle's gyration motion around the magnetic field lines, saving computation time. This new method is based on the Boris pusher, the commonly used integrator in plasma physics known for its high accuracy. To approximate the guiding centre motion, Xiao and Qin proposed to modify the initial conditions, by keeping only the velocity parallel to the magnetic field and removing the perpendicular velocity, and to consider an additional electric field.

Methods

Classical Boris-Buneman algorithm

1. Half electric acceleration

$$\vec{v}_{n+\frac{1}{2}} = \vec{v}_n + \frac{qdt}{2m}\vec{E}$$

2. Rotation vector

$$f = \frac{qdt}{2m}$$

$$\vec{v}' = \vec{v}_{n+\frac{1}{2}} + \vec{v}_{n+\frac{1}{2}} \times f\vec{B}$$

3. Magnetic rotation

$$s = \frac{2f}{1+\left(f\times\|\vec{B}\|\right)^2}$$

$$\vec{v}^+ = \vec{v}_{n+\frac{1}{2}} + \vec{v}' \times s\vec{B}$$

4. Half electric acceleration

$$\vec{v}_{n+1} = \vec{v}^+ + \frac{qdt}{2m}\vec{E}$$

5. Position update

$$\vec{r}_{n+1} = \vec{r}_n + \vec{v}_{n+1}dt$$

Initial conditions:

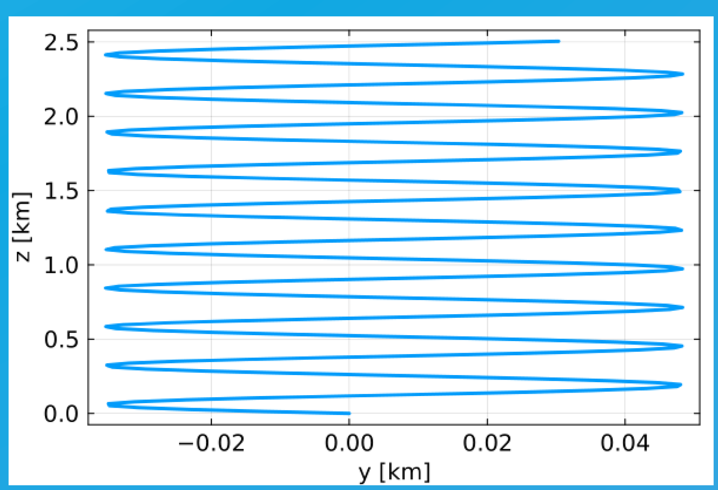
$$\vec{v} = \vec{v_0}$$
$$\vec{r} = \vec{r_0}$$

Test case: uniforme magnetic field

$$\vec{B} = B_0\vec{e_z}$$

with $B_0 = 10^{-7}$ T

Electron of kinetic energy 3 eV.



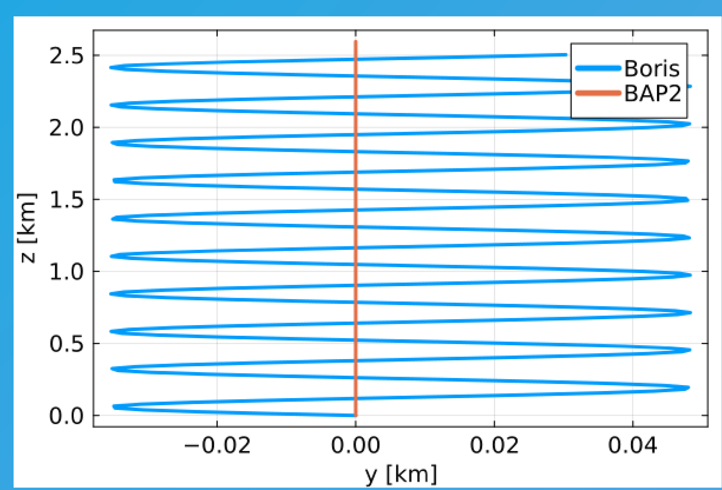
Trajectory of an electron in a constant magnetic field, with the classical Boris integrator, with a time step $dt_{Boris} = 1.8\times10^{-5}$s.

Initial conditions:

$$\vec{v} = \overrightarrow{v_{\parallel,0}}$$
$$\vec{r} = \vec{r_0}$$

With $\overrightarrow{E_{modified}} = \vec{E} - \frac{\mu_0}{q}\vec{\nabla}B$

And $\mu_0 = \frac{mv_{\perp,0}}{2B_0}$



Trajectory of an electron in a constant magnetic field, with the modified Boris integrator, with a time step $dt_{Boris} = 3.6\times10^{-3}$s.

Modified Boris-Buneman algorithm

1. Half electric acceleration

$$\vec{v}_{n+\frac{1}{2}} = \vec{v}_n + \frac{qdt}{2m}\overrightarrow{E_{modified}}$$

2. Rotation vector

$$f = \frac{qdt}{2m}$$

$$\vec{v}' = \vec{v}_{n+\frac{1}{2}} + \vec{v}_{n+\frac{1}{2}} \times f\vec{B}$$

3. Magnetic rotation

$$s = \frac{2f}{1+\left(f\times\|\vec{B}\|\right)^2}$$

$$\vec{v}^+ = \vec{v}_{n+\frac{1}{2}} + \vec{v}' \times s\vec{B}$$

4. Half electric acceleration

$$\vec{v}_{n+1} = \vec{v}^+ + \frac{qdt}{2m}\overrightarrow{E_{modified}}$$

5. Position update

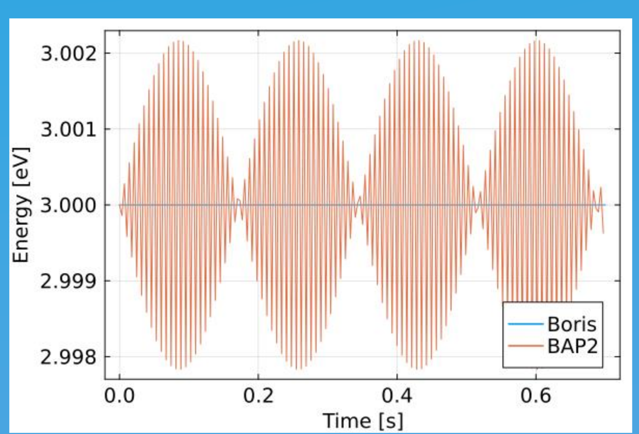
$$\vec{r}_{n+1} = \vec{r}_n + \vec{v}_{n+1}dt$$

Results

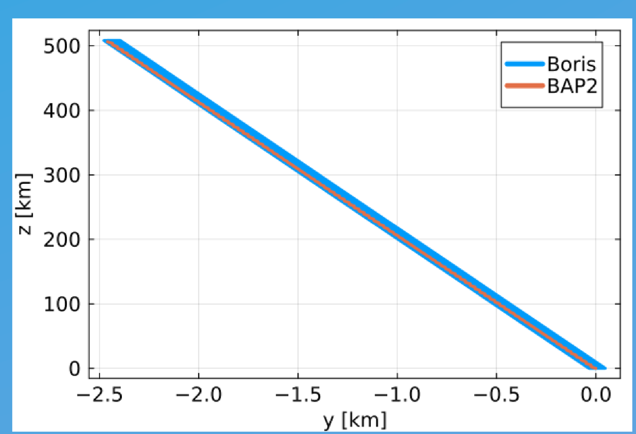
Test-case: $\vec{B}$ gradient

$$\vec{B} = B_0(1+ax)\vec{e_z} \quad \text{with } a = \frac{1}{100\rho_L} = 2.4\times10^{-4}m^{-1}$$

Electron of initial kinetic energy 3 eV, time steps $dt_{GC} = 3.5\times10^{-3}$s and $dt_{Boris} = 1.8\times10^{-5}$s



Kinetic energy of the guiding centre (orange) and the particle (blue).



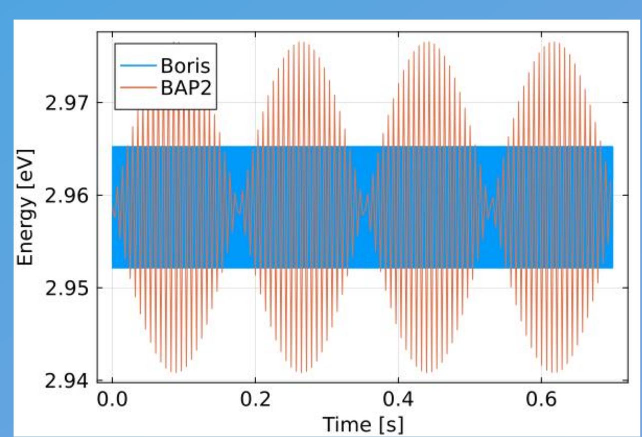
Trajectory of the guiding centre (orange) and the particle (blue) in the (x,z) plane.

Conclusion: the kinetic energy of the guiding centre is conserved and its trajectory is consistent with what is expected. The new method reproduces the drift due a magnetic gradient.

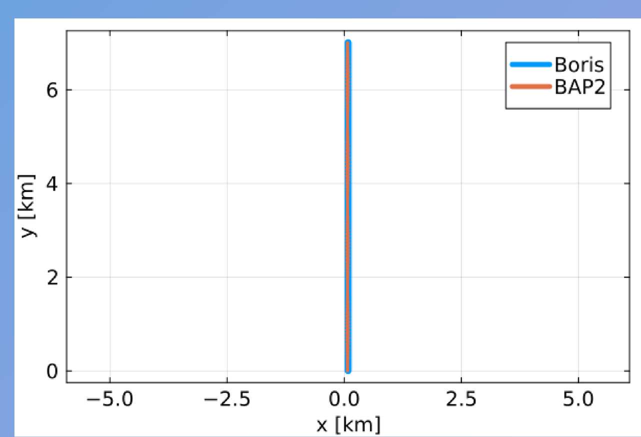
Test-case: $\vec{E}\times\vec{B}$ drift

$$\vec{E} = -0.001\vec{e}_x$$
$$\vec{B} = 10^{-7}\vec{e}_z$$

Electron of initial kinetic energy 3 eV, time steps $dt_{GC} = 3.6\times10^{-3}$ s and $dt_{Boris} = 1.8\times10^{-5}$ s



Kinetic energy of the guiding centre (orange) and the particle (blue).



Trajectory of the guiding centre (orange) and the particle (blue) in the (x,y) plane.

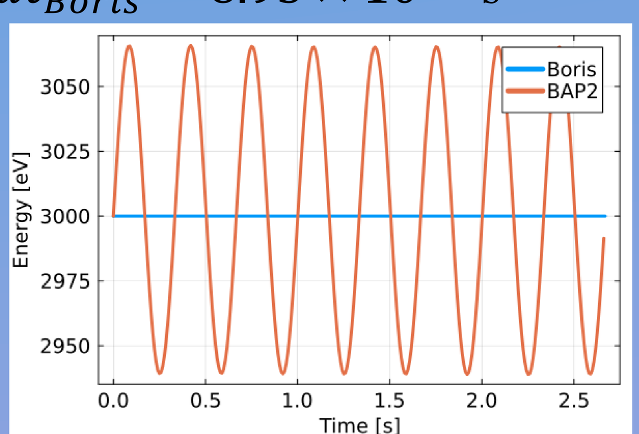
Conclusion: the kinetic energy of the guiding centre is conserved and its trajectory is consistent with what is expected. The new method reproduces the drift due the $\vec{E}\times\vec{B}$ drift.

Test-case: magnetic bottle

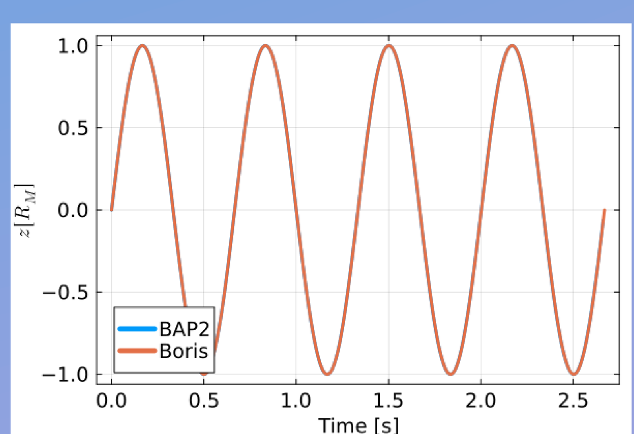
$$\vec{B} = (-B_0xz\beta)\vec{e_x} + (-B_0yz\beta)\vec{e_y} + B_0(1+\beta z^2)\vec{e_z}$$

With $\begin{cases} B_0 = 10^{-7}\,T \\ \beta = \frac{1}{R_m^2} = 1.68\times10^{-13}m^{-2} \end{cases}$ and $R_M = 2440$ km (radius of Mercury)

Electron, of initial kinetic energy 3 keV, time steps $dt_{GC} = 8.9\times10^{-3}$ s and $dt_{Boris} = 8.93\times10^{-6}$ s



Kinetic energy of the guiding centre (orange) and the particle (blue).



Position of the mirror points of the guiding centre (orange) and the particle (blue).

Conclusion: the kinetic energy of the guiding centre is conserved and its trajectory is consistent with what is expected. The new method computes correctly the mirror points in the case of a magnetic bottle.

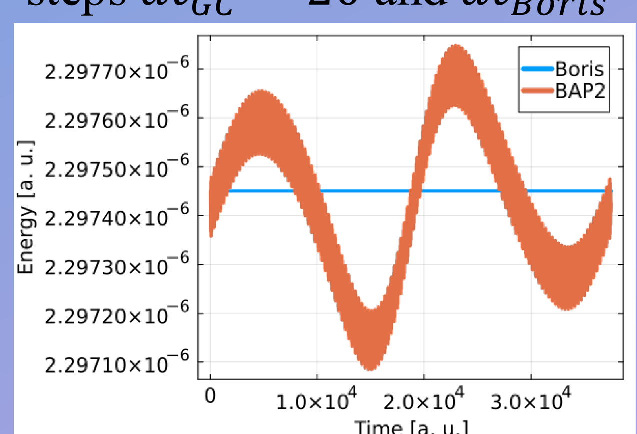
Test-case: tokamak (inspired by [2])

Normalised units $(m = 1, q = 1)$

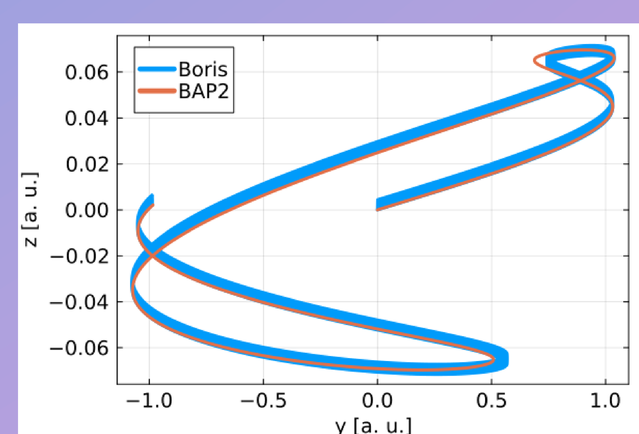
$$\vec{B} = \left(-\frac{2y+xz}{2R^2}\right)\vec{e_x} + \left(\frac{2x-yz}{2R^2}\right)\vec{e_y} + \left(\frac{R-1}{2R}\right)\vec{e_z}$$

With $R = \sqrt{x^2+y^2}$

Particle of unit mass and charge, initial kinetic energy 2.29745×10^{-6} a.u., time steps $dt_{GC} = 20$ and $dt_{Boris} = 0.2$



Kinetic energy of the guiding centre (orange) and the particle (blue).



Trajectory of the guiding centre (orange) and the particle (blue) in the (x,y) plane.

Conclusion: the kinetic energy of the guiding centre is conserved and its trajectory is consistent with what is expected.

References

[1] Slow manifolds of classical Pauli particle enable structure-preserving geometric algorithms for guiding center dynamics, Computer Physics Communications, Jianyuan Xiao, Hong Qin,

[2] Large-stepsize integrators for charged-particle dynamics over multiple time scales, Numerische Mathematik, Ernst Hairer, Christian Lubich, Yanyan Shi