Energy transfer rate estimation by an HelioSwarm-like constellation in an Hall-MHD simulation

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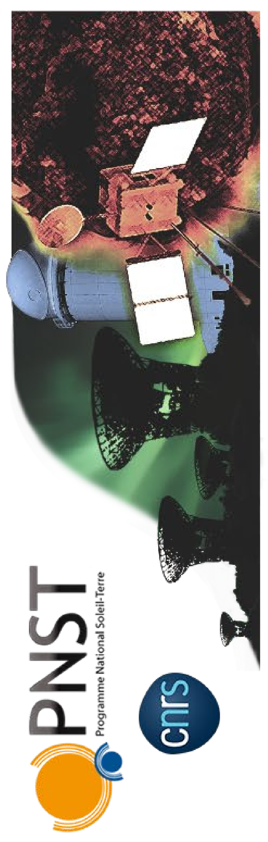


Energy transfer rate

- The energy transfer rate **ε**, or dissipation rate, is an essential parameter to assess turbulent processes in plasmas/fluids
- At intermediate scales, the inertial range, Kolmogorov's law for the energy spectrum is expressed as a function of **ε** (and *k*) as:

$$E(k) = C\varepsilon^{2/3}k^{-5/3}$$

- Estimating this parameter can be done globally in a simulation
- In space, multi-point measurements are usually too scarce in the studied range of scales and over-simplifying assumptions must be done
- Here we employ a technique proposed by *Pecora et al.* to derive an estimate from the future 9-point HelioSwarm magnetic field/plasma observations

Computing the energy transfer rate ε

Von Kármán-Howarth equation:

$$\frac{\partial}{\partial t}\left\langle|\delta z^{\pm}|^2\right\rangle = -\vec{\nabla}_\ell \cdot \left\langle \delta z^{\mp}|\delta z^{\pm}|^2\right\rangle + 2\nu\nabla_\ell^2\left\langle|\delta z^{\pm}|^2\right\rangle - 4\epsilon^{\pm}$$

- First term: Dominates at very large length scales
- Second term: Dominates at intermediate length scales = **inertial range**
- Third term: Dominates at very small length scales
- $4\epsilon^{\pm}$: The unknown: the **energy transfer rate**

With $\delta z^{\pm}(x, l) = z^{\pm}(x + l) - z^{\pm}(x)$

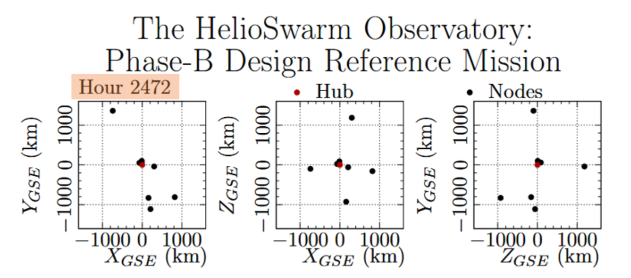
Computing the energy transfer rate ε

➔ retaining terms dominating in the intertial range, the equation becomes: $\vec{\nabla}_\ell \cdot Y^{\pm}(l) = -4\epsilon^{\pm}$

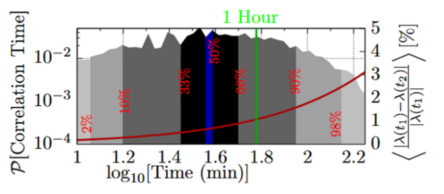
with $Y^{\pm}(l) = \left\langle \delta z^{\mp}|\delta z^{\pm}|^2\right\rangle$, the **Yaglom flux**

- ***l*** is a **lag**, i.e. a separation vector between two spacecraft (or baseline)
- The divergence must be computed in the lag space

HelioSwarm constellation

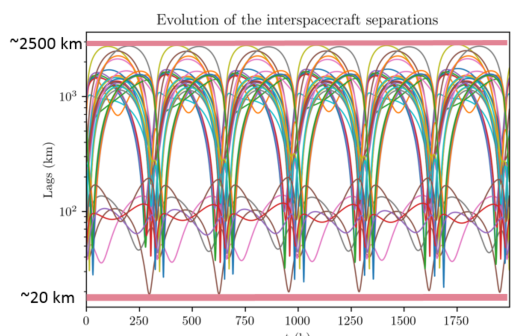


# Spacecraft	4	5	6	7	8	9	10	11
Baselines	6	10	15	21	28	36	45	55
Tetrahedra	1	5	15	35	70	126	210	330
Polyhedra	1	6	22	64	163	382	848	1816



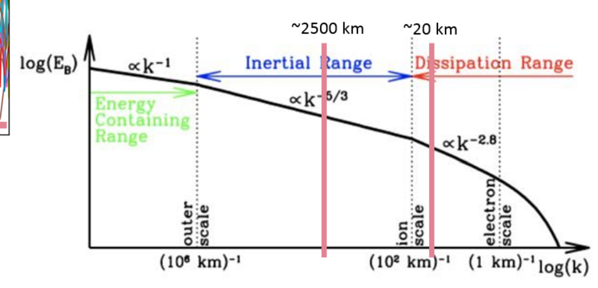
HS configuration changes of ~1% in 1h which is the typical correlation time of structures in the Solar Wind

Probing the inertial range



1 orbit = 2 weeks = 336h

- For typical solar wind conditions : c/ω_{pi} = 100 km
- With dx=0.1 c/ω_{pi} the simulation box scales as (5120 km)^3, i.e., HS spacecraft separations are contained in the box
- These separations (lags) are within the SW inertial range

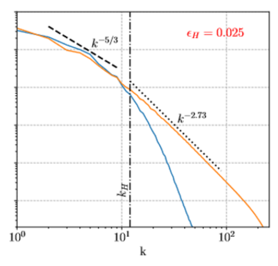


HelioSwarm science objectives: see Klein et al., 2023

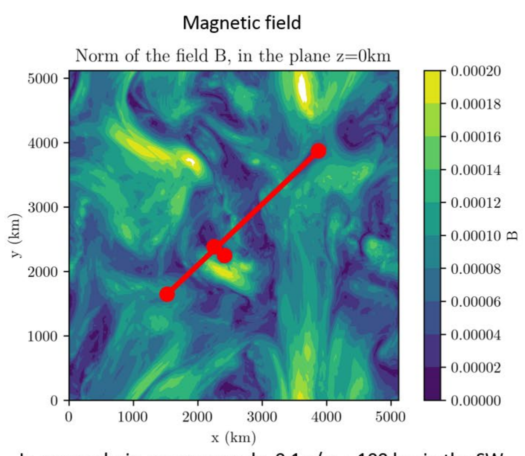
Simulation set-up

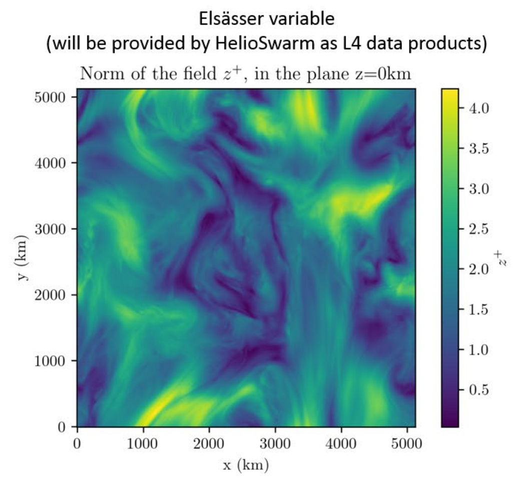
Run	N	Ma $[\times 10^{-4}]$	Re	Pr_m	ϵ_H
I	512	7.0	4400	1	0.0025
II	512	1.0	5240	1	0.01
III	512	0.625	7150	1	0.025
IV	768	0.6	6000	1	0.015

- Hall-MHD code : *Foldes et al.*, 2023
- 512^3 initialized with a Orszag-Tang vortex
- At $k>k_H$ the magnetic spectrum shows a power-law scaling that is in agreement with the $k^{-8/3}$ scaling obtained from solar wind measurements at sub-ion scales
- This code therefore enables to simulate plasmas down to sub-ion scales like HelioSwarm will be able to measure



Simulation fields at HelioSwarm scales

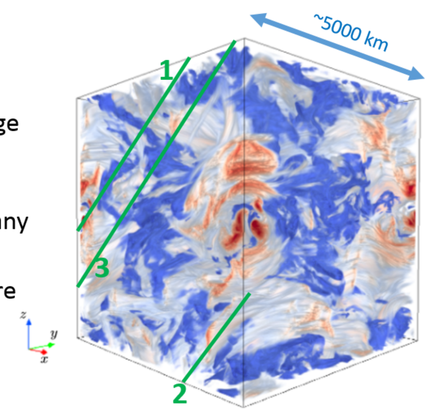




In our analysis, we assume dx=0.1 c/ω_{pi}=100 km in the SW

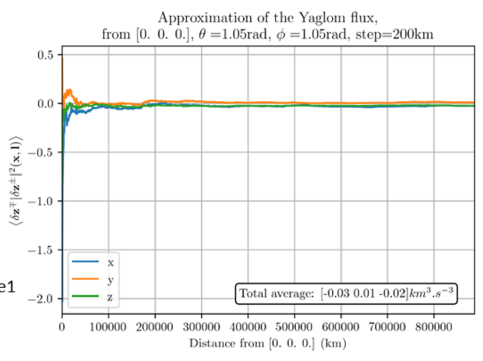
Computation of the Yaglom flux : spatial average

- $Y^{\pm}(l) = \left\langle \delta z^{\mp}|\delta z^{\pm}|^2\right\rangle$
  < . >: spatial average on a suitably large volume
- For each lag (i.e. a pair of spacecraft), we average on a straight trajectory within the periodic simulation
- Convergence of this spatial average requires many crossings of the simulation cube
- 3 example crossings are shown on the right (here on one cube side for simplicity) for a given orientation

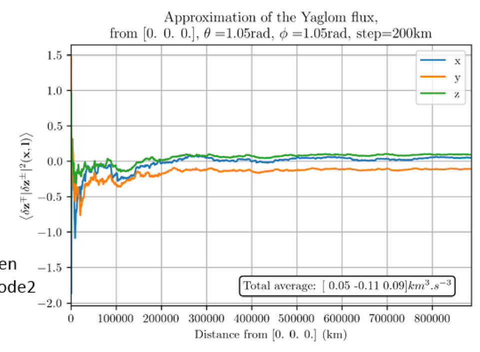


Computation of the Yaglom flux : spatial average

Lag between node0 and node1



Lag between node0 and node2

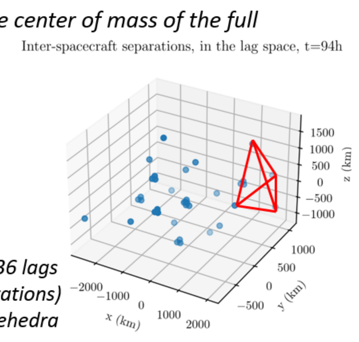


- Convergence toward a non-zero value
- The convergence distance depends on the chosen orientation
- The convergence values corresponds to the components of the vector ***Y*** (the Yaglom flux)

Computation of $\vec{\nabla}_\ell \cdot Y^{\pm}$

"Lag Polyhedral Derivative Ensemble" method (Pecora et al., 2023)

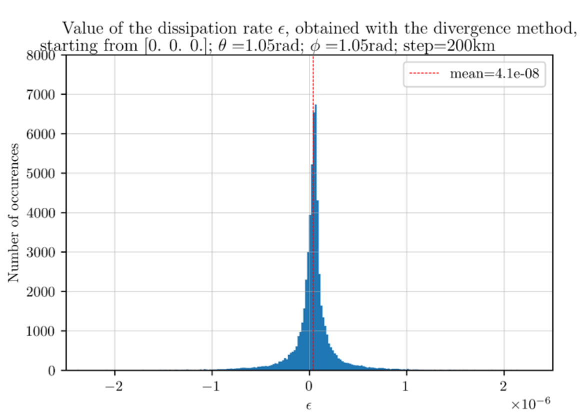
- **In real space:**
  - *Field measurement at each spacecraft*
  - *The curl/div is computed at the center of mass of a given teatrahedron*
  - *With 9 spacecraft,* $\binom{9}{4}$ *=* ***126 tetrahedra*** *are used to compute 126 values*
  - *These values are usually averaged to obtain a final value at the center of mass of the full swarm*
- **In lag space:**
  - *Similar approach is done for the computation of* $\vec{\nabla}_\ell \cdot Y^{\pm}$
  - *With 36 lags,* $\binom{36}{4}$ *=* ***58905 tetrahedra*** *can be built*
  - *Statistical distribution of* $\vec{\nabla}_\ell \cdot Y^{\pm}$ *values can then be produced*



Representation of the 36 lags (inter-spacecraft separations) with one example tetrehedra

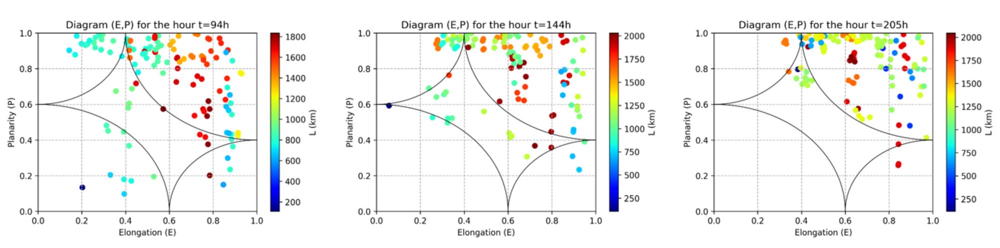
Dissipation rate calculation

- Distribution obtained with the divergence method
  - $\vec{\nabla}_\ell \cdot Y^{\pm}(l) = -4\epsilon^{\pm}$
- The histogram is composed from the 58000+ estimations of **ε**
- Mean value of **ε** dependent of the orientation of the trajectory



Selecting the best tetrahedra

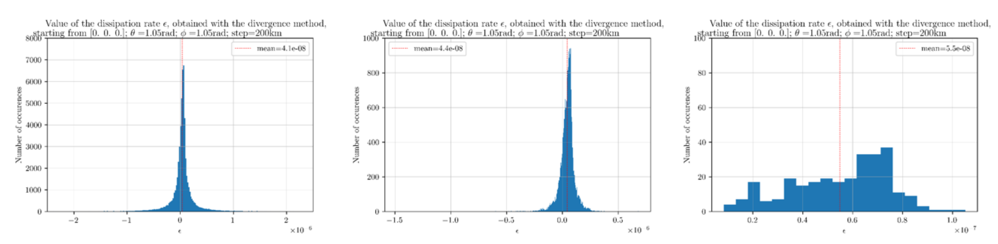
- Quality thresholds can be applied on the elongation (E) and planarity (P) values of each tetrahedral configuration (E and P are computed from the eigenvalues of the volumetric tensor)
- Typically $\sqrt{(E^2 + P^2)}$< 0.6 or 1 are used to retain only pseudo-sphere or potato type configurations known to perform well for curl/div computation
- *Similar criteria can be applied in lag space*



Example: E, P values of the 126 HelioSwarm tetrahedra at 3 different times / configurations
L is the characterictic scale of tetrahedron

Restriction on tetrahedral configurations in lag space

- $\chi = \sqrt{(E^2 + P^2)}$ < 1: 21063 tetrahedra are selected
- χ < 0.6: 276 tetrahedra are selected
- The resulting **ε** is not varying too much with reduced set of tetrahedra



All tetrahedra | $\chi < 1$ | $\chi < 0.6$

Conclusions

- The general method is a smart way to estimate **ε**
- Working with the "best" orientation, instead of averaging several of them, could be a way to recover matching values
- Averaging Yaglom flux at the tetradron scales (instead of in the whole box) should also be investigated
- HelioSwarm will be able to provide the energy transfer rate in a variety of regimes of the solar wind

References

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