A general turbulence exact law for compressible magnetized pressure-anisotropic plasmas

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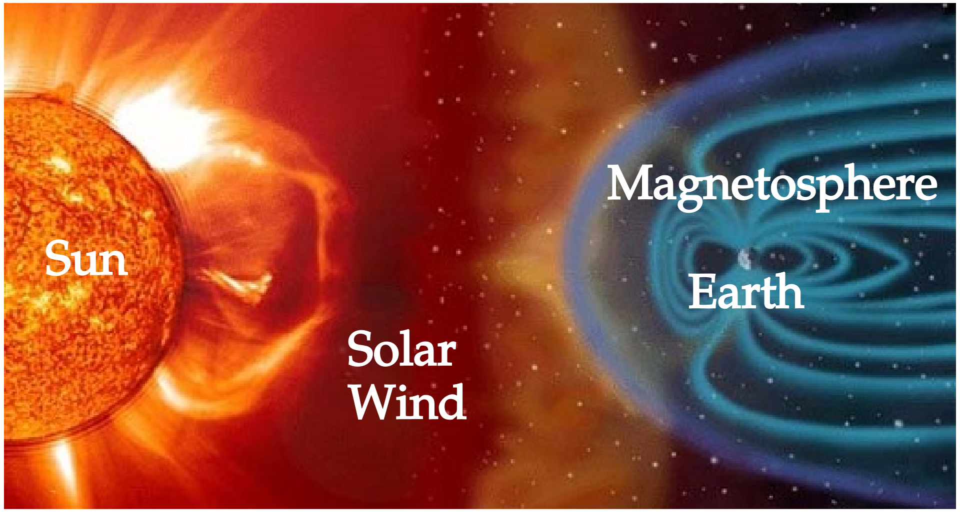
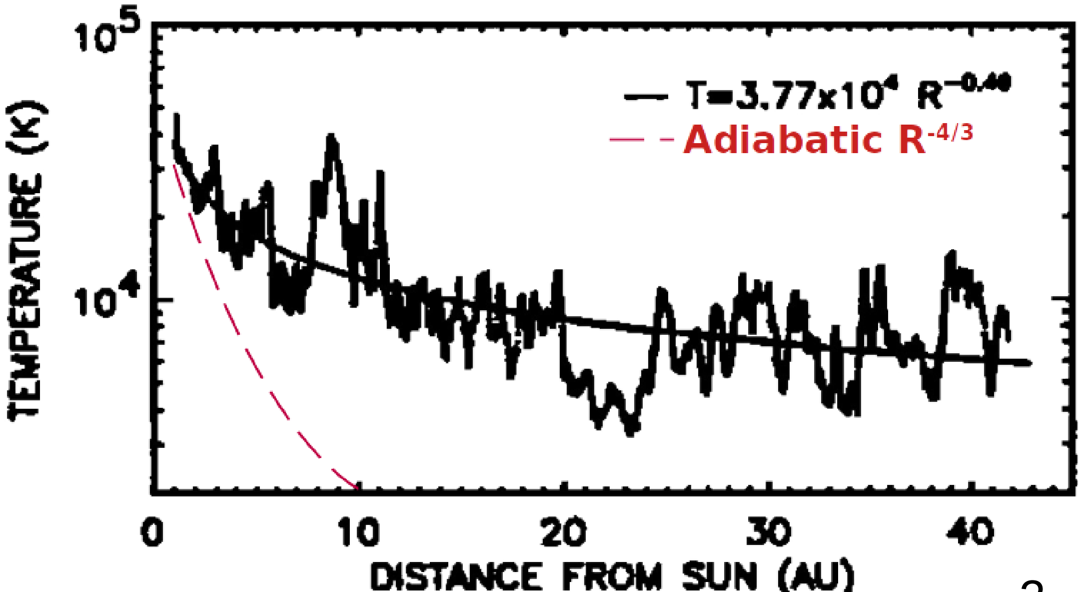
The heating issue of the Solar Wind



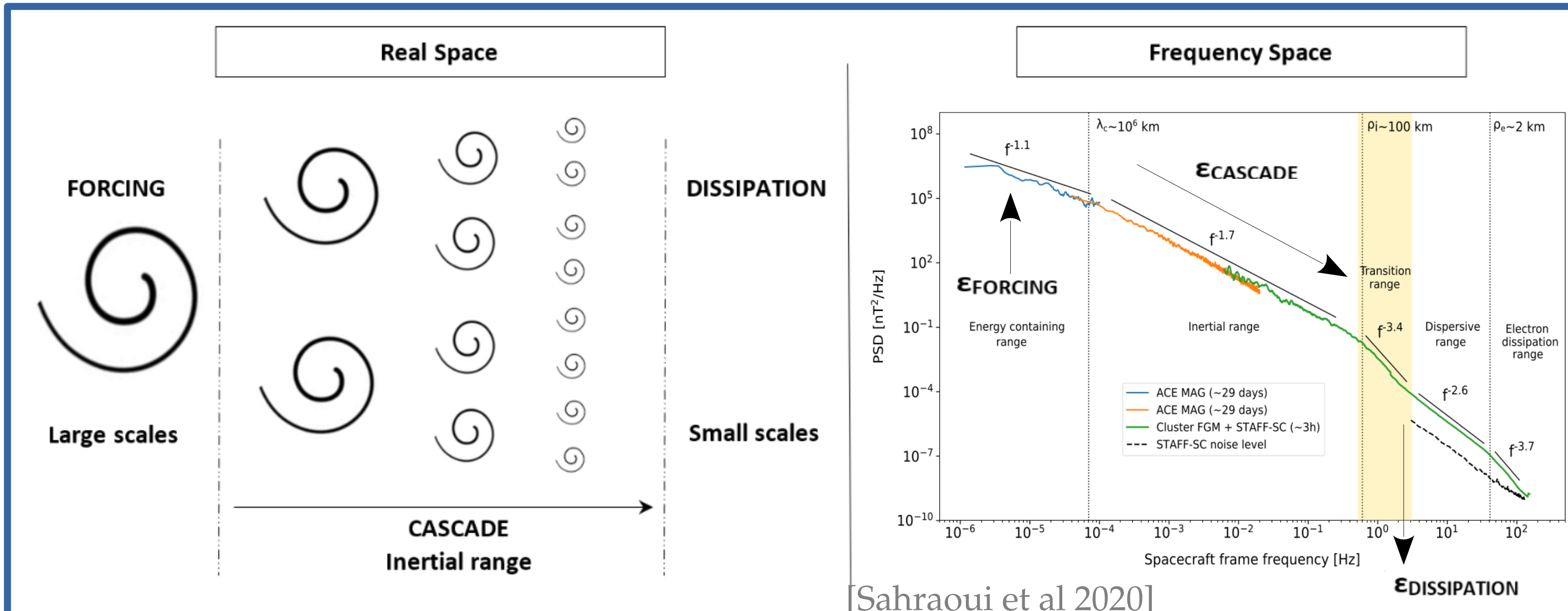
Property of the solar wind:
- Collisionless

Missions launched in the Solar Wind reported a **non-adiabatic profil of temperature.**
[Barnes 1992, Richardson 1995]

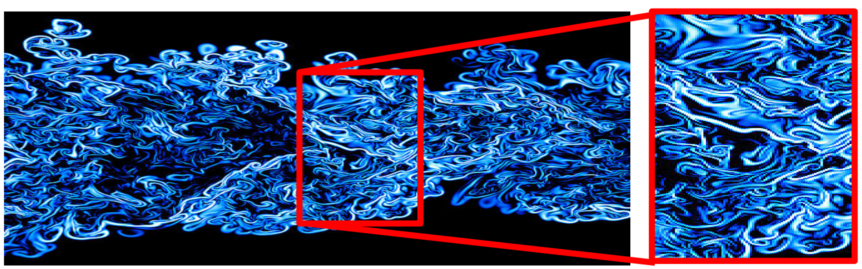
What process can keep the solar wind around 10 000K if there is no collisions?



The solution of the turbulent cascade: a transfer of energy from scale to scale until the kinetic dissipative ones at rate ε



[Sahraoui et al 2020]



[CNRS UMR 6614 CORIA and JSC]

Kolmogorov's hypothesis :

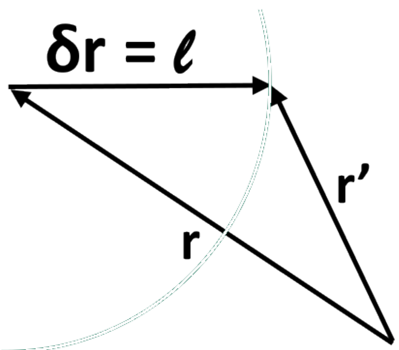
- Large scale forcing
- Small scale dissipation
- Statistical stationarity
- Statistical homogeneity
- Large Reynolds numbers

$$\varepsilon = \varepsilon_{FORCING} = \varepsilon_{CASCADE} = \varepsilon_{DISSIPATION}$$

The Kolmogorov's theory of exact laws gives a formula to evaluate the cascade rate [Kolmogorov 1941, Antonia & al 1997]

Kolmogorov hypothesis:

- Forcing at large scales
- Dissipation at small scales
- Statistical stationarity
- Statistical homogeneity
- High Reynolds numbers



Fluid model: (incompressible Navier-Stokes)

$$\partial_t \rho = -\nabla \cdot (\rho \mathbf{v}) = 0$$
$$\partial_t \mathbf{v} = -\mathbf{v} \cdot \nabla \mathbf{v} - \nabla P + \mathbf{d} + \mathbf{f}$$

Statistical temporal derivation of a correlation function between 2 points:

$$\partial_t R = \partial_t < \mathbf{v} \cdot \mathbf{v}' >= \ldots$$

... Exact Law: $$-4\frac{\varepsilon}{\rho_0} = \nabla_\ell \cdot < |\delta \mathbf{v}|^2 \delta \mathbf{v} >$$

But the Solar Wind is a magnetised and compressible fluid!

Generalisation of the method to compressible plasmas: Previous works

Version 1.0
[Banerjee & Galtier 2011,...]

Complete model : MHD equations, closure equation, explicite form of the internal energy

Statistical derivation of a correlation function:
$R = < \rho \mathbf{v} \cdot \mathbf{v}' + \rho \mathbf{v_A} \cdot \mathbf{v_A}' + \rho u' >$

Hypothesis of scale separation of Kolmogorov

EXACT LAW
for the considered model and correlation function

Previous exact laws :

- Incompressible (PP98) [Politano & Pouquet 1998]
  $\rho \propto 1 \text{ and } D_t u = 0$
- Isothermal [Banerjee & Galtier 2013] [Andrés & Sahraoui 2017]
  $p \propto \rho \text{ and } u \propto \ln(\rho/\rho_0)$
- ...

Generalisation of the method to compressible plasmas: A little trick !

Version 1.0
[Banerjee & Galtier 2011,...]

Complete model : MHD equations, closure equation, explicit form of the internal energy

Statistical derivation of a correlation function:

$$R =< \rho \mathbf{v} \cdot \mathbf{v}' + \rho \mathbf{v_A} \cdot \mathbf{v}'_{\mathbf{A}} + \rho u' >$$

Hypothesis of scale separation of Kolmogorov

EXACT LAW
for the considered model and correlation function

Version 2.0
[Simon & Sahraoui 2021 (ApJ)]

MHD equations and **the internal energy conservation equation with the isentropic hypothesis:**

$$D_t u = -\frac{p}{\rho} \nabla \cdot \mathbf{v}$$

Generalised exact law for an isotropic pressure
[Simon & Sahraoui 2021 (ApJ)]

Closure equation and the explicit form of the internal energy

The exact laws for compressible and magnetic fluids

Previous exact laws :

- Incompressible (PP98) [Politano & Pouquet 1998]

$$\rho \propto 1 \text{ and } D_t u = 0$$

- Isothermal
  [Banerjee & Galtier 2013] [Andrés & Sahraoui 2017]

$$p \propto \rho \text{ and } u \propto \ln(\rho/\rho_0)$$

Generalised exact law for an isotropic pressure
[Simon & Sahraoui 2021 (ApJ)]

$$D_t u = -\frac{p}{\rho}\nabla \cdot \mathbf{v}$$

Polytropic law

$$p \propto \rho^\gamma \text{ and } u \propto \frac{p}{\rho(\gamma - 1)}$$

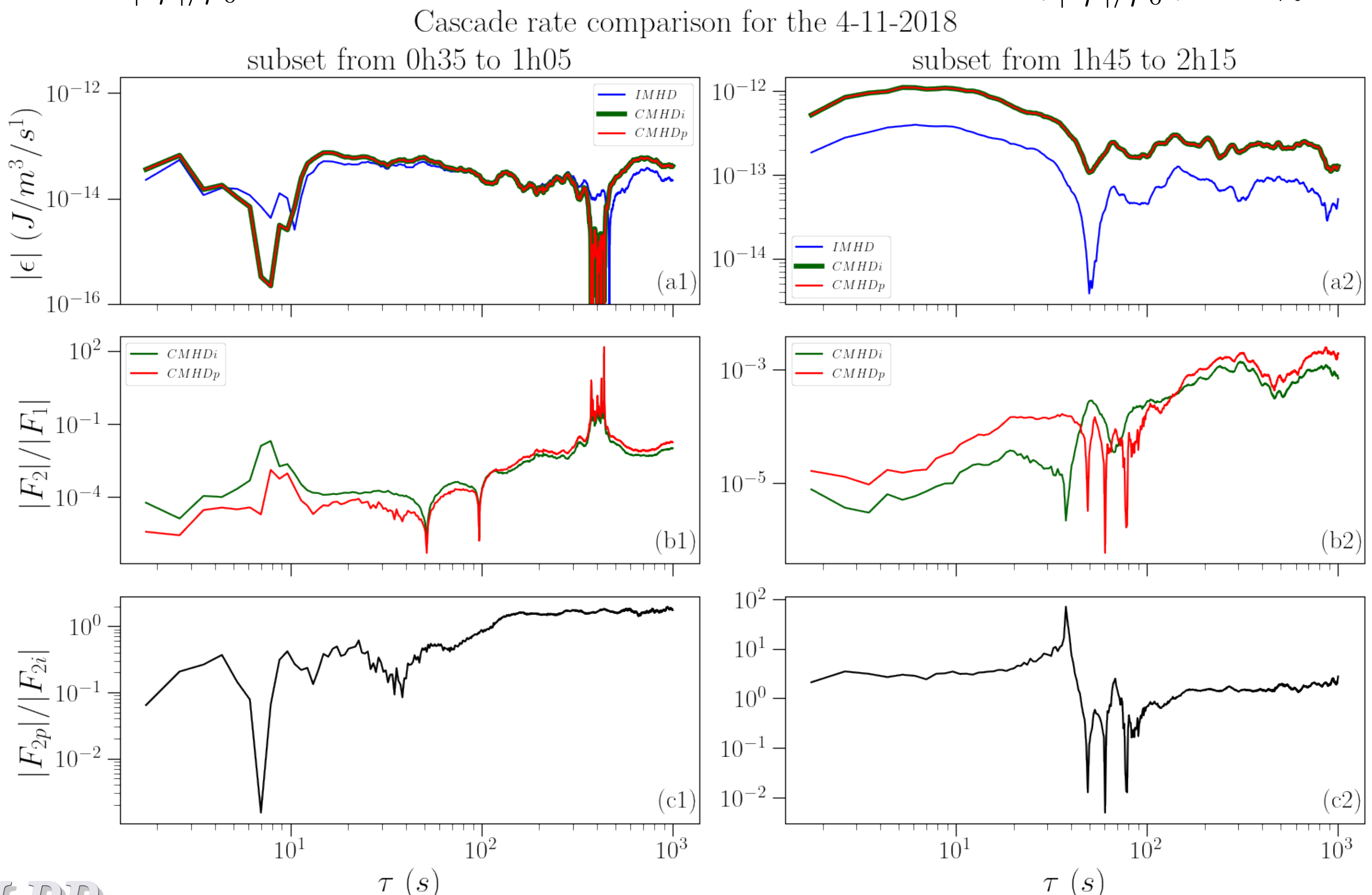
A comparative case-study of isotropic laws in Parker Solar Probe data [Simon & Sahraoui 2021 (ApJ)]

Subcase 1 : quasi-incompressible

$< |\delta\rho|/\rho_0 > \sim 8\%$

Subcase 2 : compressible

$< |\delta\rho|/\rho_0 > \sim 20\%$



The exact laws for plasmas with pressure anisotropy

Previous exact laws :

- Incompressible (PP98) [Politano & Pouquet 1998]

$$\rho \propto 1 \text{ and } D_t u = 0$$

- Isothermal [Banerjee & Galtier 2013] [Andrés & Sahraoui 2017]

$$p \propto \rho \text{ and } u \propto \ln(\rho/\rho_0)$$

Polytropic law $p \propto \rho^\gamma$ and $u \propto \dfrac{p}{\rho(\gamma-1)}$

Generalised exact law for an isotropic pressure [Simon & Sahraoui 2021 (ApJ)]

$$D_t u = -\frac{p}{\rho}\nabla\cdot\mathbf{v}$$

Generalised exact law for a tensorial pressure [Simon & Sahraoui 2022 (PRE)]

$$D_t u = -\frac{\overline{\overline{P}}}{\rho} : \nabla\mathbf{v}$$

Gyrotropic laws:

$$\overline{\overline{P}} = p_\perp \overline{\overline{I}} + (p_\| - p_\perp)\mathbf{bb} \text{ and } \rho u = \frac{1}{2}\overline{\overline{P}} : \overline{\overline{I}}$$

- **Compressible CGL (bi-adiabatic)**
- **Incompressible (Correction of PP98 depending on the pressure anisotropies)**

Gyrotropic laws: Exact law for CGL model (bi-adiabatique)

$$-4\varepsilon^{\mathrm{GYR}} = \nabla_\ell \cdot \mathcal{F}^{\mathrm{GYR}} + \mathcal{S}^{\mathrm{GYR}} + \mathcal{S}'^{\mathrm{GYR}}$$

$$\begin{cases} \mathcal{F}^{\mathrm{GYR}} = \langle \delta(\rho\mathbf{v})\cdot\delta\mathbf{v}\delta\mathbf{v} + \delta(\rho\mathbf{v_A})\cdot\delta\mathbf{v_A}\delta\mathbf{v} - \delta(\rho\mathbf{v_A})\cdot\delta\mathbf{v}\delta\mathbf{v_A} - \delta(\rho\mathbf{v})\cdot\delta\mathbf{v_A}\delta\mathbf{v_A}\rangle \\ \quad + \left\langle \delta\rho\delta\left(\frac{\mathbf{v_A}^2}{2}(\beta_\parallel[1+a_p]-1)\right)\delta\mathbf{v} \boxed{- \delta\rho\delta\left(\frac{\beta_\parallel}{2}[1-a_p]\mathbf{v_A}\mathbf{v_A}\right)\cdot\delta\mathbf{v}} \right\rangle, \\ \mathcal{S}^{\mathrm{GYR}} = \left\langle \left(\rho\mathbf{v}\cdot\delta\mathbf{v} + \frac{1}{2}\rho\mathbf{v_A}\cdot\delta\mathbf{v_A} - \frac{1}{2}\mathbf{v_A}\cdot\delta(\rho\mathbf{v_A}) + \rho\delta\left(\frac{\mathbf{v_A}^2\beta_\parallel}{2}\right)\right)\nabla'\cdot\mathbf{v}' \right\rangle \\ \quad \boxed{- \langle \rho\delta(\beta_\parallel[1-a_p]\mathbf{v_A}\mathbf{v_A}) : \nabla'\mathbf{v}'\rangle} \\ \quad + \langle(-2\rho\mathbf{v}\cdot\delta\mathbf{v_A} - \rho\mathbf{v_A}\cdot\delta\mathbf{v} + \delta(\rho\mathbf{v})\cdot\mathbf{v_A})\nabla'\cdot\mathbf{v}'_\mathbf{A}\rangle \\ \quad + \left\langle \left((\delta\rho)\frac{\mathbf{v_A}^2}{2}[a_p\beta_\parallel+1]\mathbf{v} - \rho\delta\left(\frac{\mathbf{v_A}^2}{2}[a_p\beta_\parallel+1]\right)\mathbf{v}\right)\cdot\frac{\nabla'\rho'}{\rho'} \right\rangle \\ \quad \boxed{+ \left\langle \left((\delta\rho)\frac{\beta_\parallel}{2}[1-a_p]\mathbf{v_A}\mathbf{v_A}\cdot\mathbf{v} - \rho\delta\left(\frac{\beta_\parallel}{2}[1-a_p]\mathbf{v_A}\mathbf{v_A}\right)\cdot\mathbf{v}\right)\cdot\frac{\nabla'\rho'}{\rho'} \right\rangle}, \\ \mathcal{S}'^{\mathrm{GYR}} = \text{conjugate}(\mathcal{S}^{\mathrm{GYR}}) . \end{cases}$$

With: $\beta_\parallel = \frac{p_\parallel}{p_M}$, $a_p = \frac{p_\perp}{p_\parallel} = \frac{T_\perp}{T_\parallel}$

[Simon & Sahraoui 2022 (PRE)]

Gyrotropic laws : the incompressible limit

PP98 + Correction due to pressure l'anisotropy:

$$-4\frac{\varepsilon}{\rho_0} = \nabla_\ell \cdot < (|\delta \mathbf{v}|^2 + |\delta \mathbf{v_A}|^2)\delta \mathbf{v} - 2\delta \mathbf{v} \cdot \delta \mathbf{v_A} \delta \mathbf{v_A} >$$

[Politano & Pouquet 1998]

$$+ < \delta(\beta_{\|}(1 - a_p)\mathbf{v_A}\mathbf{v_A}) : \delta(\nabla \mathbf{v}) >$$

With: $\beta_{\|} = \frac{p_{\|}}{p_M}, \; a_p = \frac{p_{\perp}}{p_{\|}} = \frac{T_{\perp}}{T_{\|}}$

Depending on the sign of this term, the pressure anisotropies can diminish or reinforce the cascade.

[Simon & Sahraoui 2022 (PRE)]

Gyrotropic laws : a link between turbulence and instabilities ?

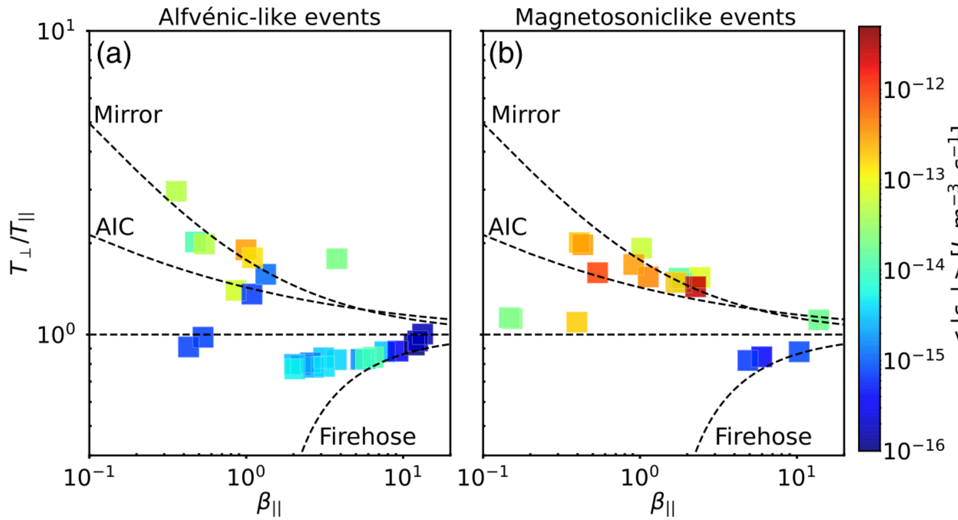
PP98 + Correction due to pressure l'anisotropy: [Simon & Sahraoui 2022 (PRE)]

$$-4\frac{\varepsilon}{\rho_0} = \underbrace{\nabla_\ell \cdot < (|\delta \mathbf{v}|^2 + |\delta \mathbf{v_A}|^2)\delta \mathbf{v} - 2\delta \mathbf{v} \cdot \delta \mathbf{v_A} \delta \mathbf{v_A} >}_{\text{PP98}} + < \delta(\beta_\parallel \underline{(1-a_p)} \mathbf{v_A}\mathbf{v_A}) : \delta(\nabla \mathbf{v}) >$$

Depending on the sign of this term, the pressure anisotropies can diminish or reinforce the cascade.

With: $\beta_\parallel = \frac{p_\parallel}{p_M}, \; a_p = \frac{p_\perp}{p_\parallel} = \frac{T_\perp}{T_\parallel}$

[Hadid et al 2018]



Conditions for firehose instability:

$a_p < 1 \Rightarrow$: Firehose instabilities

In the compressible case, the analysis is the same but, this time, it is also possible to see mirror instabilities if $a_p > 1$.

Conclusion

Results:

- An extension of the theory of exact law that gives the mean cascade rate for compressible plasmas with tensorial pressure.
- A correction of the reknown Politano and Pouquet's incompressible law due to the anisotropy of pressure.
- A potential link between linear instabilities due to pressure anisotropy and turbulence, a non-linear process.

Simon & Sahraoui, 2021
ApJ: vol 916, p49
arXiv:2105.08011

Simon & Sahraoui, 2022
PRE: in press
arXiv:2112.03601

What's next ?

- Computing the new laws in turbulence simulation data to refine our understanding of the theory.
- Look at spacecraft data to confront theory and reality.