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Magnetic Reconnection drives sub-ion turbulence:
A Coarse Graining Approach

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Overview

- Turbulence in the Solar Wind
- From a global to a local Turbulence: The Coarse Graining approach

  [D. Manzini *et al.*, submitted (2022a)]
- The Coarse Graining in practice: Magnetic Reconnection

  [D. Manzini *et al.*, submitted (2022b)]

Why Turbulence?

"Turbulence is the most important unsolved problem of classical physics."

Richard Feynman

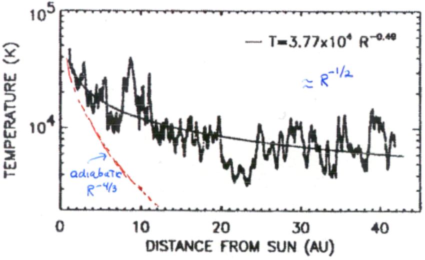


Figure: 50 days running average of the plasma temperature from Voyager 2.

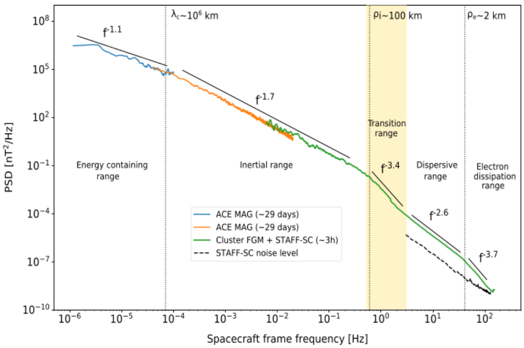
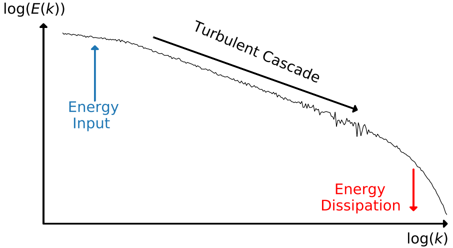


Figure: A magnetic energy spectrum measured in the solar wind. [F. Sahraoui *et al.* RMPP 2019]

Classical Theory of Hydrodynamic turbulence



Nonlinear interactions transfer energy from large to small scales at rate ε. Kolmogorov 4/5 law:

- Statistical homogeneity
- Existance of the inertial range:
  Energy input scale $\ll$ Dissipation scale

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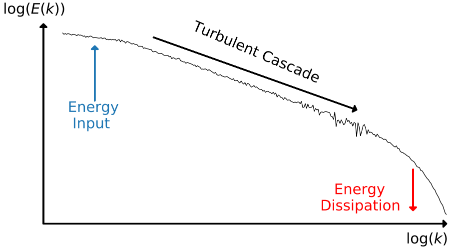

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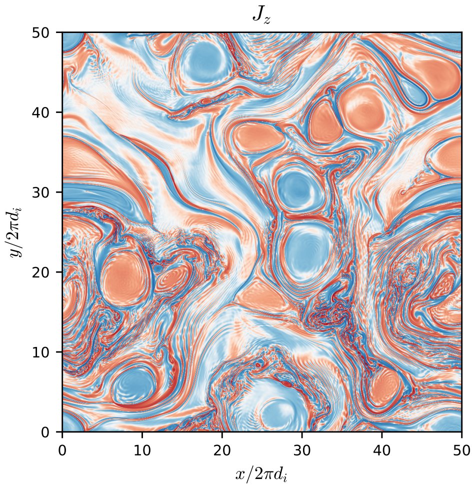
When applying to Plasma Physics:

- Statistical description: No local (in space) description
- Dissipation can occur at all scales (*e.g.* Landau Damping)

The need of Spatial Locality



Not all plasma regions are equal!

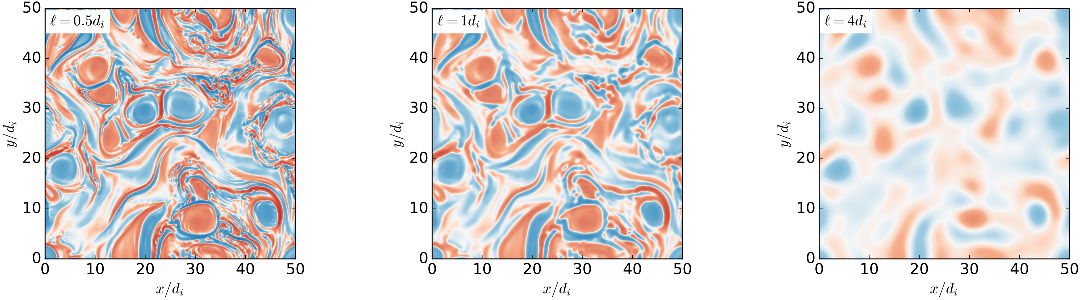
- Classical Kolmogorov Theory: Cascade rate $\epsilon \longrightarrow$ 1 Value
- In this work Local Coarse Graining: Cascade rate $\pi_\ell(\boldsymbol{x}) \longrightarrow$ Spatial dependance

From global to local cascade: The CG Approach

Normalized filtering function $G_\ell(\boldsymbol{x})$ with variance $\sim \ell^2$. *e.g* $G_\ell(\boldsymbol{x}) = \frac{e^{-|\boldsymbol{x}|^2/(2\ell^2)}}{\sqrt{2\pi\ell^2}}$

The coarse-graining operation is a local average of the field:

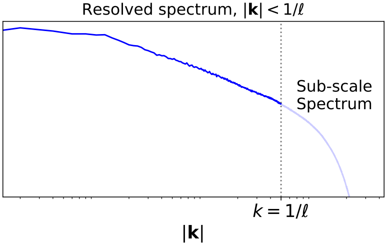
$$\bar{\mathbf{u}}_\ell = \mathbf{u} * G_\ell = \int d^3x' \mathbf{u}(\boldsymbol{x}') G_\ell(\boldsymbol{x} - \boldsymbol{x}')$$



The CG approach in Fourier space

$$\bar{\mathbf{u}}_\ell = \mathbf{u} * G_\ell \longrightarrow \widehat{\bar{\mathbf{u}}} = \hat{\mathbf{u}} \cdot \hat{G}_\ell \propto \hat{\mathbf{u}}(\boldsymbol{k}) e^{-2\pi\ell^2|\boldsymbol{k}|^2}$$

exponential cut-off at $k = 1/\ell$



For each choice of ℓ we divide the range of scales in

- the large scales (resolved)
- The small scales (un-resolved)

Introducing: $\mathbf{b} = \boldsymbol{B}/\sqrt{\mu_0\rho_0}$ $\quad \boldsymbol{j} = \nabla \times \mathbf{b}$ $\quad P = p + |\mathbf{b}|^2/2$
we can write equations for the large scale fields $\bar{\mathbf{u}}_\ell, \bar{\mathbf{b}}_\ell \ldots$

The CG incompressible HMHD equations

The equations for the CG fields at a scale ℓ read:

$$\begin{aligned}\partial_t \bar{\mathbf{u}} &= -(\bar{\mathbf{u}} \cdot \nabla)\bar{\mathbf{u}} + (\bar{\mathbf{b}} \cdot \nabla)\bar{\mathbf{b}} - \nabla \bar{P} + \nabla \cdot \boldsymbol{\tau} + \nu \nabla^2 \bar{\mathbf{u}} \\ \partial_t \bar{\mathbf{b}} &= \nabla \times (\bar{\mathbf{u}} \times \bar{\mathbf{b}}) - d_i \nabla \times (\bar{\boldsymbol{j}} \times \bar{\mathbf{b}}) + \nabla \times \boldsymbol{\mathcal{E}} + \eta \nabla^2 \bar{\mathbf{b}} \\ \nabla \cdot \bar{\mathbf{b}} &= 0 \quad \nabla \cdot \bar{\mathbf{u}} = 0\end{aligned} \tag{1}$$

Equations describe the large scale ($|\boldsymbol{k}| \lesssim 1/\ell$) fields.

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Same shape as non filtered equations with the influence of the non resolved terms (small scales, $|\boldsymbol{k}| \gtrsim 1/\ell$):

$$\boldsymbol{\tau}_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j - (\overline{b_i b_j} - \bar{b}_i \bar{b}_j) \qquad \boldsymbol{\mathcal{E}} = \overline{\mathbf{u} \times \mathbf{b}} - \bar{\mathbf{u}} \times \bar{\mathbf{b}} - d_i(\overline{\boldsymbol{j} \times \mathbf{b}} - \bar{\boldsymbol{j}} \times \bar{\mathbf{b}})$$

CG Energy equations

The Goal:

Write equations for the large scale energy and study the cross-scale energy flux.

$$\partial_t \left(\frac{|\bar{\mathbf{u}}_\ell|^2 + |\bar{\mathbf{b}}_\ell|^2}{2} \right) = -\nabla \cdot \mathcal{J}_\ell - \pi_\ell(\boldsymbol{x}) - \nu |\bar{\boldsymbol{w}}_\ell|^2 - \eta |\bar{\boldsymbol{j}}_\ell|^2 \tag{2}$$

- Spatiall advection by the term $\nabla \cdot \mathcal{J}_\ell$
- Large scale dissipation $-\nu |\bar{\boldsymbol{w}}_\ell|^2 - \eta |\bar{\boldsymbol{j}}_\ell|^2$
- $\pi_\ell = -\nabla \bar{\mathbf{u}}_\ell : \tau_\ell - \bar{\boldsymbol{j}}_\ell \cdot \boldsymbol{\mathcal{E}}_\ell$ is the amount of energy (at position $\boldsymbol{x}$) going from large to small scales

The CG energy transfer $\pi_\ell(\boldsymbol{x})$

Localized (in space) energy transfer: $\pi_\ell(\boldsymbol{x})$ is the transfer across scale ℓ at position $\boldsymbol{x}$.

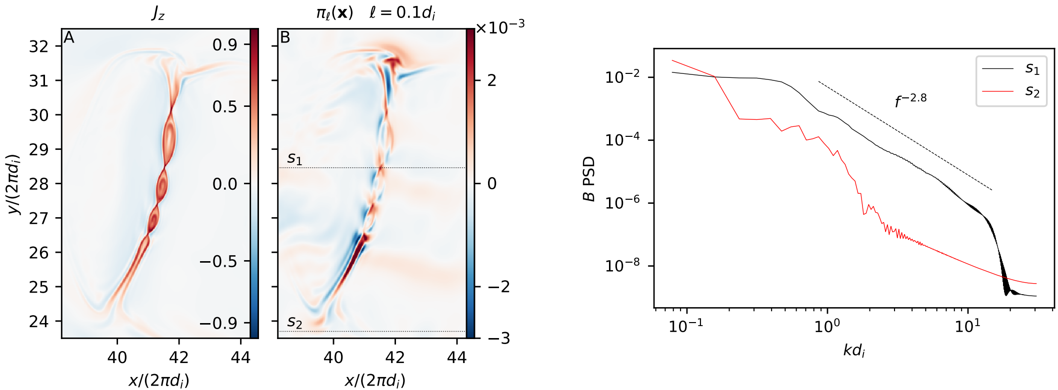
$\pi_\ell(\boldsymbol{x}) > 0$: Direct cascade, $\pi_\ell(\boldsymbol{x}) < 0$: Inverse cascade.

$\pi_\ell(\boldsymbol{x})$ function of both the scale ℓ and the position $\boldsymbol{x}$

- Fixed $\ell \longrightarrow$ Which spatial regions are involved in energy transfer at fixed scale ℓ.
- Fixed $\boldsymbol{x} \longrightarrow$ At what scales the non-linear transfer is effective.

Application to Mangetic Reconnection

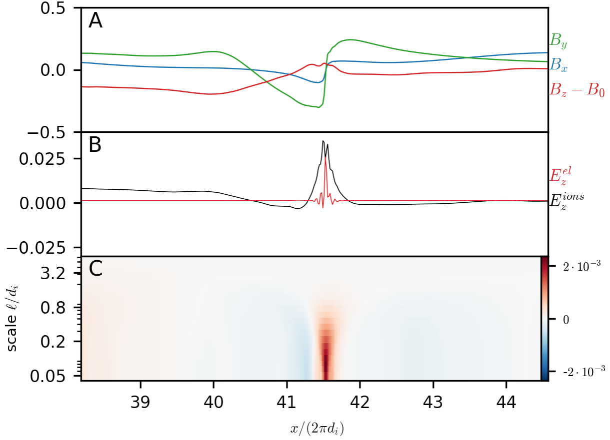
CG application to MR: Spatial features



- cascade rate at $\ell = d_i/10$ shows both positive and negative values
- Localized (positive) cascade rate at reconnecting location
- X-points and IDR involved in the creation of small scale turbuent spectrum

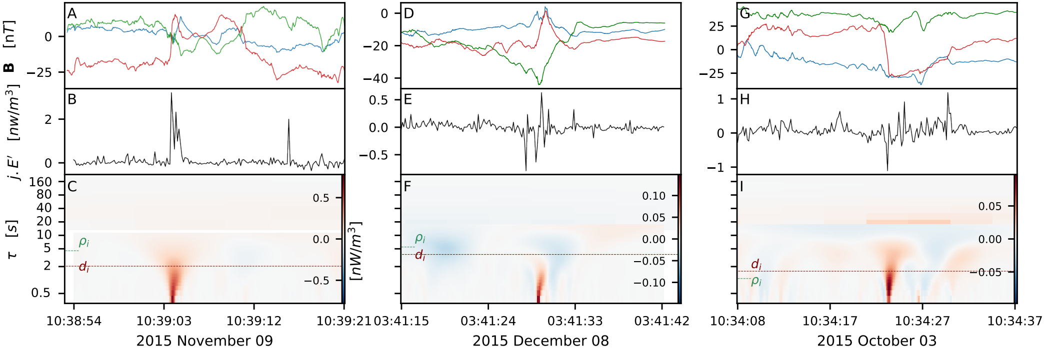
MR in HVM simulations: Scale features

1D cut across reconnecting CS



- turbulent energy cascade starts at $\ell \sim d_i$
- spatial extension comparable with EDR size

Magnetosheath Reconnection - MMS



Conclusions

- The Coarse Graining approach as a tool to describe localized energy cascade
- Magnetic reconnection is highly involved cross-scale energy transfer
- Magnetic reconnection drives sub-ion scale turbulence

[Manzini *et al.*, submitted (2022b)]

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Thanks for the attention!